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# OPTIMUM THREE MODULI SET FOR ANY WORD BIT LENGTH N AND $I = 0, 1, 2$

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**Abstract**—In Residue Number System (RNS) the most important factor is the choice of optimum moduli set. The most used three moduli set is  $M_0 = (2^n - 1, 2^n, 2^n + 1)$ , but it is only optimum when the word bit length N is exactly divisible by 3. A new three moduli set is proposed  $(2^n - 1, 2^{n+i}, 2^n + 1)$ , where  $i = |N|_3$ . It is shown to have the same dynamic range efficiency as  $M_0$  for any N. This new Moduli set is implemented using MRC II and CRT. The CRT implementation is shown to have same architecture for any i. The hardware consists of  $6n$  FA,  $2n + 1 + i$  inverters with propagation delay of  $2n + 2$ .

**Keywords**—Residue to Binary Converters, Residue Number System, Chinese Remainder Theorem II.

## I. INTRODUCTION

RNS has attracted many researchers for the past few decades that involve extensive computation in the field of signal processing such as fast Fourier Transform (FFT), Image Processing, and Digital Filters [1],[2]. High speed and less complexity of hardware requires the large number to be split into smaller numbers so that instead of one large computation several small computations can be executed in parallel which reduces the load and time of calculation. Thus, choice of proper moduli set could greatly make the design efficient and cost effective. RNS moduli sets which are discovered till now are mostly in the form of power of 2's. Further the moduli set could be a group of 3, 4 or 5 depending upon the complexity and the dynamic range of system.

In this paper we proposed a new three moduli set which overcomes the limitation of conventional and mostly commonly used three moduli set  $(2^n - 1, 2^n, 2^n + 1)$  [3][4]. In our work we considered three cases:

Let N be the Word Bit Length.

1. If  $|N|_3 = 0$  then use moduli set  $M_0 = (2^n - 1, 2^n, 2^n + 1)$  which is same as the most popular moduli set.
2. If  $|N|_3 = 1$  then use moduli set  $M_1 = (2^n - 1, 2^{n+1}, 2^n + 1)$ .
3. If  $|N|_3 = 2$  then use moduli set  $M_2 = (2^n - 1, 2^{n+2}, 2^n + 1)$ .

Once the correct moduli set is chosen, the next target it to convert RNS into binary system so that RNS based computations could be interfaced with the real-world binary computers. In our work we implemented the ideas of M. Akkal and P. Siy [5] using MRC-II and Ahmad Hiasat and Andraos

Sweidan [6] using CRT. In the later sections we have also concluded that our new moduli set is better in terms of efficiency.

## II. BACKGROUND

There are different three moduli set that are proposed in the literatures, but the most popular three moduli set is  $(2^n - 1, 2^n, 2^n + 1)$  [3]. Some other three moduli sets are  $(2n-1, 2n, 2n+1)$ [7],  $(2n, 2n+1, 2n+2)$ [8],  $(2^n - 1, 2^n + 1, 2^{2n} + 1)$ [11],  $(2^{k-1} - 1, 2^k - 1, 2^k)$  [9],  $(2^n - 1, 2^n + 1, 2^{2n+1})$  [10],  $(2^n, 2^n - 1, 2^{2n-1} - 1)$  [12],  $(2^{n-1}, 2^n - 1, 2^n + 1)$ [6] have been proposed. Even all of the above-mentioned moduli sets have been proposed and claims some advantages, but no study yet has been carried out to compare these moduli sets.

**A. Characteristics of Moduli Set:** Dynamic Range and Dynamic Range Efficiency can be used to compare moduli set.

1) The **Dynamic Range** is defined as the product of moduli in a given set.

$$M = \prod_{i=1}^3 m_i \quad (1)$$

2) **Dynamic range efficiency 'D<sub>eff</sub>'** is the ratio of dynamic range of the moduli set to dynamic range of word bit length.

$$D_{\text{eff}} = M / 2^N \quad (2)$$

The following example will illustrate how those above-mentioned parameters are calculated. Let us consider word bit length N = 9 bit. According to  $M_0$  the three moduli set is  $2^n - 1, 2^n, 2^n + 1$ .

$$\text{where } n \text{ is calculated as } n = N / 3 = 9 / 3 = 3 \quad (3)$$

If we substitute the above calculated value of n as 3 then the three moduli sets are:

$$m_1 = 2^3 - 1 = 7, m_2 = 2^3 = 8, m_3 = 2^3 + 1 = 9$$

**a. Dynamic range** according to (1) is given as:

$$= (2^n) * (2^n - 1) * (2^n + 1) = 2^{3n} - 2^n = 504. \quad (4)$$

**b. Dynamic Range Efficiency (D<sub>eff</sub>)** according to (2) is:

$$= (2^{3n} - 2^n) / 2^{3n} = 1 - 1 / 2^{2n}$$

$$= 1 - 1/2^{2*3} = 1 - 1/64 = .99 \text{ or } 99\% (5)$$

The efficiency of  $M_0$  is the reason why  $M_0$  is the most cited 3 moduli set. It will be shown that efficiency disappears if  $|N|_3 \neq 0$ .

**1. Case1:  $|N|_3 = 1$**

For  $N = 10$ , when divided by 3 gives us remainder 1 and

$$n = (N - 1) / 3$$

Value of  $n$  when calculated comes out to be 3 and the modulus set is 7, 8, 9.

**Dynamic range** according to (1) is **504**.

**Dynamic range efficiency ( $D_{\text{eff}}$ )**, when calculated according to (2)

$$D_{\text{eff}} = 504 / 1024 \cong 50\% \quad (6)$$

This 3 Moduli set can only represents 50% of the desired dynamic range.

**2. Case2:  $|N|_3 = 2$**

For  $N = 11$ , which when divided by 3 gives remainder 2. So the value of  $n$  is calculated using and  $n = (N - 2) / 3$

Calculating the value of  $n$  yields 3 and the modulus set is 7, 8, 9.

**Dynamic range** according to (1) comes out to be **504**.

**Dynamic range efficiency ( $D_{\text{eff}}$ )**, when calculated according to (2):

$$D_{\text{eff}} = 504 / 2048 \cong 25\% \quad (7)$$

Again this 3 moduli set can only represents 25% of the desired dynamic range.

### III. CORRECTED MODULI SET

According to the most popular moduli set the three moduli set is  $M_0 = 2^n - 1, 2^n, 2^n + 1$ . But as we discussed in above section that it is no longer efficient if  $N$  is not exactly divisible by 3. In order to overcome these shortcomings, we had approached the problem in different way and thus come up with a unique approach of solving this problem. We took two cases again.

1. When bit word length 'N' divided by 3 gives remainder 1
2. When bit word length 'N' divided by 3 gives remainder 2.

The modified moduli set for:

**1. When  $|N|_3 = 1$**

In this case we had increased the central modulus by one so that our new moduli set looks like  $M_1 = 2^n - 1, 2^{n+1}, 2^n + 1$ . We had further verified that this is the most efficient moduli set.

**Example 3.1:** Let us consider  $N = 10$ . The moduli set according to our modified formula is;  $M_1 = 2^n - 1, 2^{n+1}, 2^n + 1$  is 7, 16, 9.

**(a) Dynamic range:** As defined in the section 2.1, the dynamic range is the product of all the modulus in the moduli set and is given as:

$$M = \prod_{i=1}^3 m_i = (2^n - 1) * (2^{n+1}) * (2^n + 1) \\ = (2^{n+1}) * (2^{2n} - 1) = 2^{3n+1} - 2^{n+1} \quad (8)$$

$$N = 3n + 1:$$

$$n = (10 - 1) / 3 = 3$$

Substituting the value of  $n$  in (8)

$$M = (2^{3*3+1} - 2^{3+1}) = (1024 - 16) = 1008 \quad (9)$$

**(b) Dynamic Range Efficiency  $D_{\text{eff}}$**  is defined as the ratio of dynamic range ( $M$ ) to dynamic range of bit word length.

$$D_{\text{eff}} = M / 2^N = \frac{(2^{3n+1} - 2^{n+1})}{2^{3n+1}} \\ = 1 - \frac{1}{2^{2n}} = 1 - \frac{1}{64} = .99 \text{ or } 99\% \quad (10)$$

This is same as equation (5).

**1. When  $|N|_3 = 2$**

In this case we had increased the central modulus by two so that our new moduli set  $M_2 = 2^n - 1, 2^{n+2}, 2^n + 1$ . In order to verify that this is most efficient moduli set we took an example as shown below.

**Example 3.2:** Let us consider  $N = 11$ . The moduli set according to the modified formula is;  $M_2 = 2^n - 1, 2^{n+2}, 2^n + 1$  is 7, 32, 9.

**(a) Dynamic range:** The dynamic range is the product of all the modulus in the moduli set and is given as:

$$M = \prod_{i=1}^3 m_i = (2^{n+2}) * (2^{2n} - 1) = 2^{3n+2} - 2^{n+2} \quad (11)$$

$$N = 3n + 2;$$

$$n = (11 - 2) / 3 = 3$$

And if we substitute the above value of  $n$  in (11) we get

$$M = (2^{3*3+2} - 2^{3+2}) = (2048 - 32) = 2016 \quad (12)$$

**(b) Dynamic Range Efficiency  $D_{\text{eff}}$**  is defined as the ratio of dynamic range ( $M$ ) to dynamic range of bit word length.

$$D_{\text{eff}} = M / 2^N = \frac{(2^{3n+2} - 2^{n+2})}{2^{3n+2}} = 1 - \frac{1}{2^{2n}} = 1 - \frac{1}{64} = .99 \text{ or } 99\% \quad (13)$$

This is same as equation (5).

The results can be summarized in the following theorem.



**Theorem 1:** For any word bit length  $N$ , the optimized 3 moduli set are given below:

1. If  $|N|_3 = 0$  so  $N = 3n$ , then Moduli set is given as  $M_0 = (2^n - 1, 2^n, 2^{n+1})$ .
2. If  $|N|_3 = 1$  so  $N = 3n + 1$  then Moduli set is given as  $M_1 = (2^n - 1, 2^{n+1}, 2^{n+1})$ .
3. If  $|N|_3 = 2$  so  $N = 3n + 2$  then Moduli set is given as  $M_2 = (2^n - 1, 2^{n+2}, 2^{n+1})$ .

With  $D_{\text{eff}} = 1 - \frac{1}{2^{2n}}$

IV. RNS TO BINARY CONVERTER

Once we have the correct moduli set, the next target undoubtedly should be to ascertain the appropriate RNS to Binary converter and then plugging the RNS arithmetic to

Binary world. For that we have followed the work of M.Akkal and P. Siy [5] using MRC-II and Ahmad Hiasat and Andraos Sweidan [6] using CRT.

**A. RNS to Binary Converter using MRC II**

According to MRC II [5] number  $X$  is represented as:

$$X_i = a_i M_{i-1} + X_{i-1} \text{ for } 2 \leq i \leq n; n \geq 2 \quad (14)$$

Where  $a_i$  is the MRC coefficient,  $n$  is the number of moduli,  $X_1 = a_1 = r_1$ ,  $X_n = X$  and  $M_{i-1} = \prod_{j=1}^{i-1} m_j$ . Coefficient  $a_i$  can be computed using small look up tables. These tables are labeled as  $a_i$ . For  $n$  moduli set the number of look-up tables required are  $n - 1$ , with a size of  $m_i \log_2 m_i$ , where  $m_i$  is the  $i^{\text{th}}$  moduli. The functional block diagram for calculating  $X_i$  is shown in figure 1.

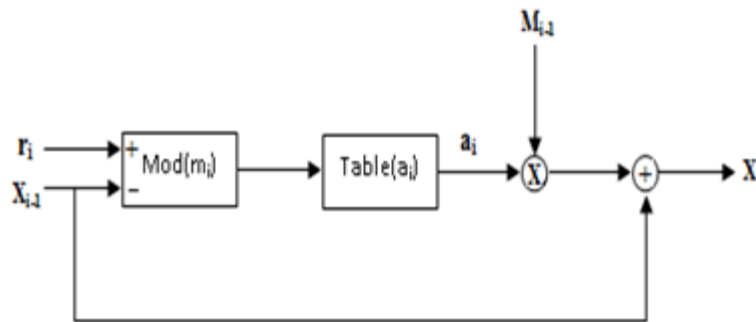


Fig. 1: Functional block diagram for  $i^{\text{th}}$  module.

Table index =  $|r_i - X_{i-1}|_{m_i}$  for  $i \geq 2$  and table content of coefficients are given as;

$$a_i; \text{ such that } |r_i - X_{i-1}|_{m_i} = |a_i \cdot M_{i-1}|_{m_i} \quad 0 \leq a_i < m_i$$

Our main goal is to keep the look up tables small and their size should also be small. This could be realized by choosing  $m_1$  to be the largest one. Order of other moduli does not affect the memory size.

**B. Implementation of three Moduli Set**

Let us implement the concept of 3 moduli set for any bit word length  $N$ . The implementation holds good for all the three cases i.e.  $|N|_3 = 0$ ,  $|N|_3 = 1$ ,  $|N|_3 = 2$ . We will discuss for  $|N|_3 = 1$  in detail. Figure 2 shows the conversion process, where 2 modules are required to calculate the MRC coefficients.

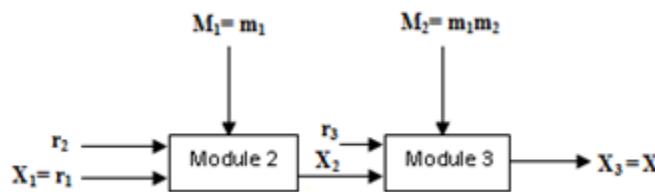


Fig.2: 3 Moduli set

**Implementation.**

**For  $|N|_3 = 1$**

According to formulae for the generation of moduli sets the moduli set is given as

$$2^n - 1, 2^{n+1}, 2^n + 1 \text{ and } N = 3n + 1 \quad (15)$$

Again we took  $N = 10$ . So by using equation (15),  $n$  comes out to be 3 and the moduli set should be 7, 16, 9. As is the

requirement of MRC II [5], the largest modulus should be designated as  $m_1$ . Thus the moduli set are  $m_1 = 16$ ,  $m_2 = 7$  and  $m_3 = 9$ .

In our case  $X$  is represented as:

$$X = X_3 = a_3 m_1 m_2 + a_2 m_1 + a_1 \quad (16)$$



From (14)  $X_1 = a_1 = r_1$  and  $M_1 = m_1$ .

$$|r_2 - X_1|_7 = |a_2 \cdot 16|_7 \tag{18}$$

And

$$|r_2 - X_1|_{m_2} = |a_2 \cdot M_1|_{m_2}; \tag{17}$$

Substituting the values of  $M_1$  and  $m_2$ , equation (17) becomes:

In order to calculate the values of  $a_2$  to build the look-up table, we need to know all the possible values of  $|r_2 - X_1|_7$  and in our case the values are  $\{0, 1, 2, 3, 4, 5, 6\}$ . We do not need to know all the individual values

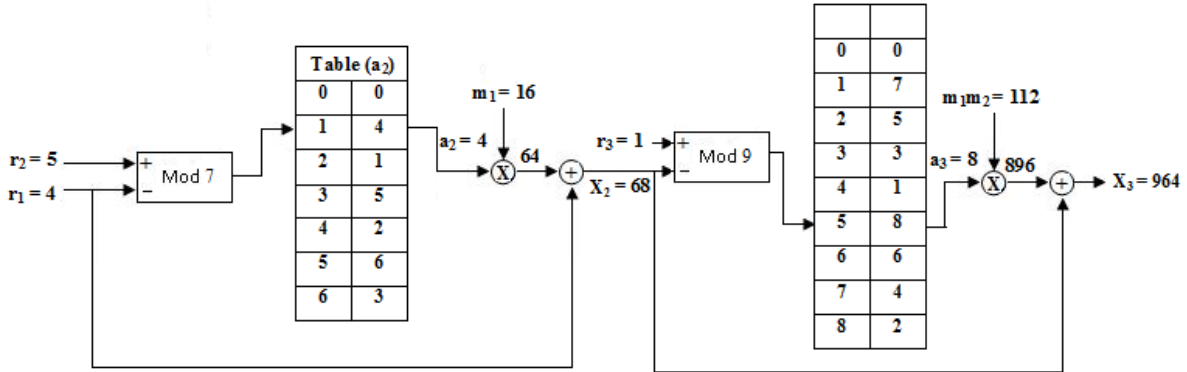


Fig. 3: MRC II Implementation flow diagram

of  $r_2$  or  $X_1$ . Coefficient  $a_2$  is calculated using (18)

Successive values of  $a_2$  can be calculated and hence look-up table is constructed as shown in table 1.

$$|r_2 - X_1|_7 = 0 = |a_2 \cdot 16|_7 \Rightarrow a_2 = 0;$$

$$|r_2 - X_1|_7 = 1 = |a_2 \cdot 16|_7 \Rightarrow a_2 = 4$$

Table (a <sub>2</sub> )		Table (a <sub>3</sub> )	
$ r_2 - X_1 _7$	$a_2$	$ r_3 - X_2 _9$	$a_3$
0	0	0	0
1	4	1	7
2	1	2	5
3	5	3	3
4	2	4	1
5	6	5	8
6	3	6	6
		7	4
		8	2

Table 1: Two tables for moduli set  $\{m_1, m_2, m_3\} = 16, 7, 9$

Coefficient  $a_3$  shown above in the table are calculated in the same manner as coefficient  $a_2$  are calculated. For the construction of table (a<sub>3</sub>):

$$|r_3 - X_2|_{m_3} = |a_3 \cdot M_2|_{m_3} \tag{19}$$

where  $M_2 = m_1 \cdot m_2 = 16 \cdot 7 = 112$  and  $m_3 = 9$ ;

$$0 \leq r_3 < m_3 \text{ and } 0 \leq X_2 < M_2$$

Substituting the values of  $M_2$  and  $m_3$ , equation (19) becomes

$$|r_3 - X_2|_9 = |a_3 \cdot 112|_9$$

And

$$|r_3 - X_2|_9 = 0 = |a_3 \cdot 112|_9 \Rightarrow a_3 = 0$$

$$|r_3 - X_2|_9 = 1 = |a_3 \cdot 112|_9 \Rightarrow a_3 = 7$$

The complete flow diagram for 3 moduli set implementation is shown in figure 3. Here we have shown the conversion of RNS number (4, 5, 1) with respect to  $(m_1, m_2, m_3 = 16, 7, 9)$  to binary number 964. From look up table 1, value of  $a_2 = 4$  corresponding to  $|r_2 - X_1|_7 = 1$ . Using formula from equation

(14) the value of  $X_2$  comes out to be 68. Further the value of  $a_3$  from look up table is 8 corresponding to  $|r_3 - X_2|_7 = 5$ . Again using (14) the value of  $X_3$  is 964 as is shown as the final output in figure 3.

a. Hardware Implementation of MRC II: The look up tables used in our work is all small tables. In 3 moduli set, only two look up tables are required. Further we can see in table 1

that the values of  $|r_2 - X_1|_7$  ranges from 0 to 6, so three bits (x y z) are required to represent those numbers. Similarly, coefficients of  $a_2$  ranges from 0 to 6, so again three bits (a b c) are required to represent them as well. As shown below in look up table design xyz and abc are taken as input and output variables respectively. In case of  $a_3$  there would be 4 input and 4 output variables.

**Truth table and Circuit Implementation of  $a_2$ :**

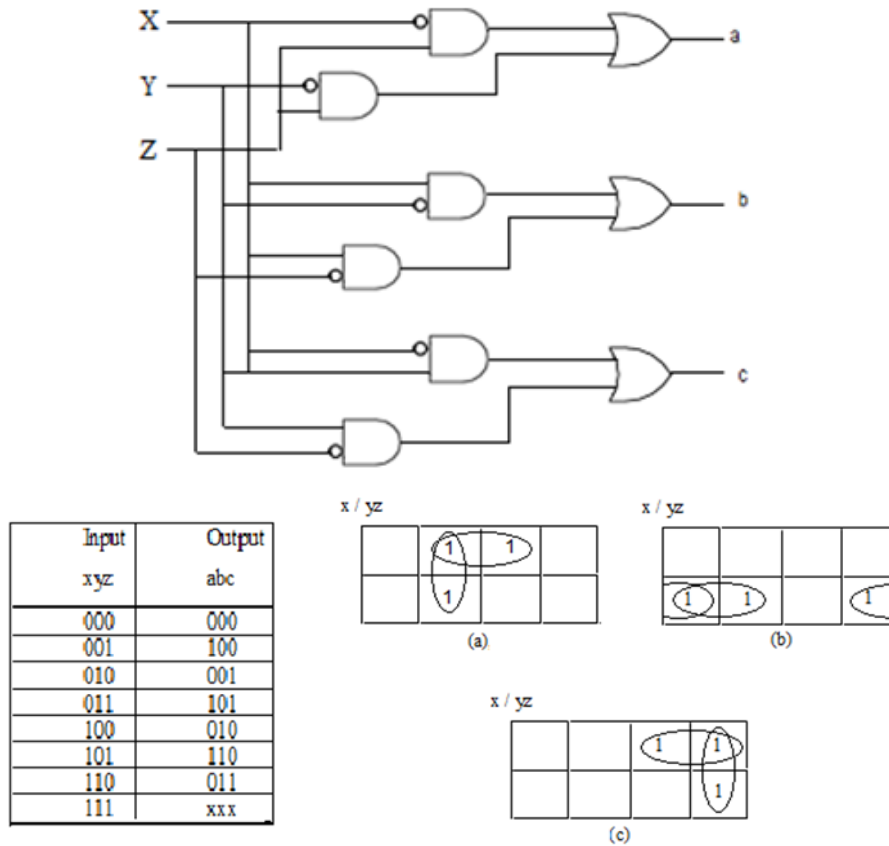


Fig.4: Look up Table Design for  $a_2$

The outputs of (a), (b) and (c) are:  
 $a = X'Z + Y'Z$      $b = X Y' + X Z'$      $c = X'Y + Y Z'$

Similarly, we can implement the other two moduli sets i.e.  $M_0$  and  $M_2$  for remainder zero and remainder 2

respectively. The coefficients  $a_2$  and  $a_3$  for the above said moduli set are calculated and shown in the table 2 and 3. Table 2 shows the coefficients for  $M_2$  and table 3 is for  $M_0$ .



Table (a <sub>2</sub> )		Table (a <sub>3</sub> )	
$ r_2 - X_1 _7$	$a_2$	$ r_3 - X_2 _9$	$a_3$
0	0	0	0
1	2	1	8
2	4	2	7
3	6	3	6
4	1	4	5
5	3	5	4
6	5	6	3
		7	2
		8	1

**Table 2 : Two tables for moduli set  $\{m_1, m_2, m_3\} = 32, 7, 9$**

Table (a <sub>2</sub> )		Table (a <sub>3</sub> )	
$ r_2 - X_1 _8$	$a_2$	$ r_3 - X_2 _7$	$a_3$
0	0	0	0
1	1	1	4
2	2	2	1
3	3	3	5
4	4	4	2
5	5	5	6
6	6	6	3
7	7		

**Table 3: Two tables for moduli set  $\{m_1, m_2, m_3\} = 9, 8, 7$**

**C. RNS to Binary Converter using CRT.**

In this section we will implement our three-modulus set using CRT [6]. Again, we have three cases i.e. we have one modulus set in which N is exactly divisible by 3 and the other two where we get remainder 1 or 2 when N is not exactly divisible by 3. In this implementation we will discuss one case where we get remainder 1 when N is divided by 3.

**For  $|N|_3 = 1$**

The moduli set in this case is  $M_1 = (m_1, m_2, m_3) = (2^{n+1}, 2^n - 1, 2^n + 1)$ . Nonnegative number  $X = (x_1, x_2, x_3)$  where  $X_i = |X|_{m_i}$  and i ranges from 1 to 3. The dynamic range is:

$$M = \prod_{i=1}^3 m_i = (2^{n+1}) * (2^n - 1) * (2^n + 1)$$

According to Chinese remainder theorem (CRT) number X is defined as:

$$X = \sum_{i=1}^N \hat{m}_i \left\langle \frac{1}{\hat{m}_i} \right\rangle_{m_i} X_i - MN(X) \quad (21)$$

For 3 moduli set CRT equation is reduced to:

$$X = \hat{m}_1 \left\langle \frac{1}{\hat{m}_1} \right\rangle_{m_1} X_1 + \hat{m}_2 \left\langle \frac{1}{\hat{m}_2} \right\rangle_{m_2} X_2 + \hat{m}_3 \left\langle \frac{1}{\hat{m}_3} \right\rangle_{m_3} X_3 - MN(X) \quad (22)$$

where  $\hat{m}_i = \frac{M}{m_i}$ ;  $\left\langle \frac{1}{\hat{m}_i} \right\rangle_{m_i}$  is multiplicative inverse of  $\hat{m}_i$  and  $N(X)$  is an integer.

- $\hat{m}_i$  is defined as the ratio of Dynamic range M and i<sup>th</sup> Moduli set [6]. where i range from 1 to 3. Consequently  $\hat{m}_1 = (2^n - 1) * (2^n + 1) = (2^{2n} - 1)$ ;  $\hat{m}_2 = (2^{n+1}) * (2^n + 1)$  and  $\hat{m}_3 = (2^{n+1}) * (2^n - 1)$  (23)



- **Multiplicative inverses**  $\langle \frac{1}{\hat{m}_i} \rangle_{m_i}$  are given in the following theorem:

**Theorem 2:** For the moduli set  $(m_1, m_2, m_3) = (2^{n+1}, 2^n - 1, 2^n + 1)$  the multiplicative inverses are:

$$\langle \frac{1}{\hat{m}_1} \rangle_{m_1} = \langle -1 \rangle_{m_1}$$

$$\langle \frac{1}{\hat{m}_2} \rangle_{m_2} = \langle 2^{n-2} \rangle_{m_2}$$

And  $\langle \frac{1}{\hat{m}_3} \rangle_{m_3} = \langle -2^{n-2} \rangle_{m_3}$

Proof: We will show that for  $i = (1, 2, 3)$   $\langle \langle \hat{m}_i \rangle_{m_i} \langle \frac{1}{\hat{m}_i} \rangle_{m_i} \rangle_{m_i} = 1$

For  $i = 1$ ;  $\langle \hat{m}_1 \rangle_{m_1} = \langle 2^{2n} - 1 \rangle_{2^{n+1}} = \langle -1 \rangle_{2^{n+1}}$

$$\langle \frac{1}{\hat{m}_1} \rangle_{m_1} = \langle -1 \rangle_{2^{n+1}} \quad (24)$$

$$\langle \langle \hat{m}_1 \rangle_{m_1} \langle \frac{1}{\hat{m}_1} \rangle_{m_1} \rangle_{m_1} = \langle (-1)(-1) \rangle_{2^{n+1}} = 1$$

For  $i = 2$

$$\langle \hat{m}_2 \rangle_{m_2} = \langle (2^{n+1})(2^n + 1) \rangle_{2^{n-1}} = \langle 4 \rangle_{2^{n-1}}$$

$$\langle \frac{1}{\hat{m}_2} \rangle_{m_2} = \langle 2^{n-2} \rangle_{2^{n-1}} \quad (25)$$

$$\langle \langle \hat{m}_2 \rangle_{m_2} \langle \frac{1}{\hat{m}_2} \rangle_{m_2} \rangle_{m_2} = \langle (4)(2^{n-2}) \rangle_{2^{n-1}}$$

$$= \langle (2^n) \rangle_{2^{n-1}} = \langle (2^n + 1 - 1) \rangle_{2^{n-1}} = 1$$

For  $i = 3$

$$\langle \hat{m}_3 \rangle_{m_3} = \langle (2^{n+1})(2^n - 1) \rangle_{2^{n+1}} = \langle 4 \rangle_{2^{n+1}}$$

$$\langle \frac{1}{\hat{m}_3} \rangle_{m_3} = \langle -2^{n-2} \rangle_{2^{n+1}} \quad (26)$$

$$\langle \langle \hat{m}_3 \rangle_{m_3} \langle \frac{1}{\hat{m}_3} \rangle_{m_3} \rangle_{m_3} = \langle (4)(-2^{n-2}) \rangle_{2^{n+1}}$$

$$= \langle (-2^n) \rangle_{2^{n+1}} = \langle (2^n + 1 - 2^n) \rangle_{2^{n+1}} = 1$$

We have all the parameters that are required in equation (22). Substituting we get

$$X = - (2^{2n} - 1)X_1 + [(2^{n+1})(2^n + 1)(2^{n-2})]X_2 + [(2^{n+1})(2^n - 1)(-2^{n-2})] - (2^{n+1})(2^n - 1)(2^n + 1)N(X) \quad (27)$$

Divide equation (27) by  $2^{n+1}$  and take the floor value of both sides.

$$\left\lfloor \frac{X}{2^{n+1}} \right\rfloor = \lfloor -2^{n-1} \rfloor X_1 + \lfloor 2^{2n-2} + 2^{n-2} \rfloor X_2 + \lfloor -2^{2n-2} + 2^{n-2} \rfloor X_3 - (2^{2n} - 1)N(X) \quad (28)$$

In order to eliminate the term  $(2^{2n} - 1)N(X)$  take modulus  $2^{2n} - 1$  both sides of (28)

$$\left\lfloor \frac{X}{2^{n+1}} \right\rfloor = \langle -2^{n-1} X_1 \rangle_{2^{2n-1}} + \langle (2^{2n-2} + 2^{n-2}) X_2 \rangle_{2^{2n-1}} + \langle (-2^{2n-2} + 2^{n-2}) X_3 \rangle_{2^{2n-1}} \quad (29)$$

X is represented by equation:

$$X = \left\lfloor \frac{X}{2^{n+1}} \right\rfloor 2^{n+1} + X_1 \quad (30)$$

Equation (29) is fractionated in parts as  $u_1, u_2$  and  $u_3$  and is expressed as:

$$\left\lfloor \frac{X}{2^{n+1}} \right\rfloor = \langle u_1 + u_2 + u_3 \rangle_{2^{2n-1}} \quad (31)$$

where  $u_1 = \langle -2^{n-1} X_1 \rangle_{2^{2n-1}}$ ;  
 $u_2 = \langle (2^{2n-2} + 2^{n-2}) X_2 \rangle_{2^{2n-1}}$   
 $u_3 = \langle (-2^{2n-2} + 2^{n-2}) X_3 \rangle_{2^{2n-1}}$

In order to realize (31) we need to obtain the simplified expression of  $u_1, u_2$  and  $u_3$  by taking modulus of respective terms.

**Evaluation of  $u_1, u_2$  and  $u_3$ :** For the Binary representation of  $X_i, x_{ij}$  represents  $j^{\text{th}}$  bit and its weight is expressed as  $2^j$ . The complement is given as  $\bar{x}_{ij}$ .

- **Evaluating  $u_1$ :**  $X_1$  is  $n + 1$  bit and its  $2n$  bit representation is:

$$X_1 = \overbrace{00 \dots 00}^{n-1} \overbrace{\bar{x}_{1n} x_{1(n-1)} \dots x_{11} x_{10}}^{n+1}$$

$$-X_1 = \overbrace{11 \dots 11}^{n-1} \overbrace{\bar{x}_{1n} \bar{x}_{1(n-1)} \dots \bar{x}_{11} \bar{x}_{10}}^{n+1}$$

For  $u_1$  multiply above equation with  $2^{n-1}$  and take modulus  $2^{2n} - 1$ , so that equation looks like:

$$u_1 = \langle -2^{n-1} X_1 \rangle_{2^{2n-1}} = \overbrace{\bar{x}_{1n} \bar{x}_{1(n-1)} \dots \bar{x}_{11} \bar{x}_{10}}^{n+1} \overbrace{11 \dots 11}^{n-1} \quad (32)$$

- **Evaluating  $u_2$ :**  $X_2$  is  $n$  bit. For simplification we split  $u_2$  into two parts;

$u'_2 = \langle 2^{2n-2} X_2 \rangle_{2^{2n-1}}$  and  $u''_2 = \langle 2^{n-2} X_2 \rangle_{2^{2n-1}}$   $2n$  bit representation of  $X_2$  is:

$$X_2 = \overbrace{00 \dots 00}^{n+1} \overbrace{x_{2(n-1)} x_{2(n-2)} \dots x_{21} x_{20}}^{n-1}$$

$$u'_2 = \langle 2^{2n-2} X_2 \rangle_{2^{2n-1}} = \overbrace{x_{21} x_{20} 0 0 \dots 0}^n \overbrace{0 0 x_{2(n-1)} x_{2(n-2)} \dots}^n$$

$$u''_2 = \langle 2^{n-2} X_2 \rangle_{2^{2n-1}} = \overbrace{0 0 x_{2(n-1)} x_{2(n-2)} \dots}^n \overbrace{x_{21} x_{20} 0 0 \dots}^n \quad (34)$$

(33)

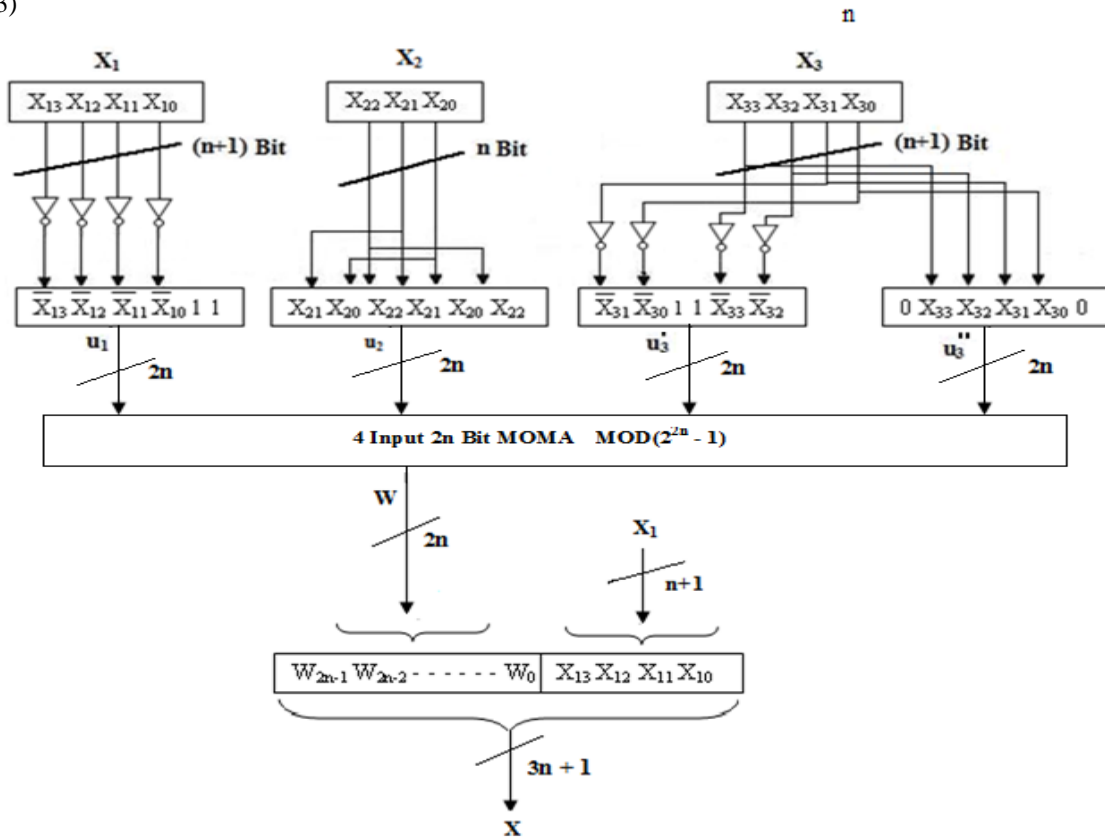


Fig. 5: Functional Block Diagram

$$u_2 = u'_2 + u''_2 = \langle (2^{2n-2} + 2^{n-2}) X_2 \rangle_{2^{2n-1}}$$

$$u_2 = \overbrace{x_{21} x_{20} x_{2(n-1)} x_{2(n-2)} \dots}^n \overbrace{x_{21} x_{20} x_{2(n-1)} x_{2(n-2)} \dots}^n$$

(35)

• **Evaluating  $u_3$ :**  $X_3$  is  $n+1$  bit. Again, for simplification we split  $u_3$  in two parts;

$$u'_3 = \langle -2^{2n-2} X_3 \rangle_{2^{2n-1}} \text{ and } u''_3 = \langle 2^{n-2} X_3 \rangle_{2^{2n-1}}$$

2n bit representation of  $X_3$  is:

$$X_3 = \overbrace{0 0 \dots 0 0}^{n-1} \overbrace{x_{3n} x_{3(n-1)} \dots x_{31} x_{30}}^{n+1}$$

$$-X_3 = \overbrace{1 1 \dots 1 1}^{n-1} \overbrace{\bar{x}_{3n} \bar{x}_{3(n-1)} \dots \bar{x}_{31} \bar{x}_{30}}^{n+1}$$

$$u'_3 = \langle -2^{2n-2} X_3 \rangle_{2^{2n-1}} = \overbrace{\bar{x}_{31} \bar{x}_{30} 1 1 \dots 1 1}^n \overbrace{\bar{x}_{3n} \bar{x}_{3(n-1)} \dots}^n$$

(36)

$$u''_3 = \langle 2^{n-2} X_3 \rangle_{2^{2n-1}} = \overbrace{0 x_{3n} x_{3(n-1)} \dots x_{32} x_{31} x_{30}}^n \overbrace{0 0 \dots 0}^n$$

(37)

$$u_3 = u'_3 + u''_3 = \langle (-2^{2n-2} + 2^{n-2}) X_3 \rangle_{2^{2n-1}}$$

In  $u_3$ , both  $u'_3$  and  $u''_3$  are 2n bits and are overlapping. Thus we took  $u'_3$  and  $u''_3$  separately. Now we have simplified  $u_1$ ,  $u_2$ ,  $u_3$ ,  $u_3$  so that these parameters could be incorporated in equation (31) for the realization. Eventually we need to get the value of  $X$  from equation (30). The parameter  $\left\lfloor \frac{X}{2^{n+1}} \right\rfloor$  is multiplied by  $2^{n+1}$  or a shifting  $\left\lfloor \frac{X}{2^{n+1}} \right\rfloor$  by  $n+1$  bits and adding  $X_1$ , an  $n+1$  bit number. The complete functional block

Table 4: Expression for  $U_1, U_2$  and  $U_3$

$ N _3=0$	$ N _3=1$	$ N _3=2$
$u_1 = \langle -2^n X_1 \rangle_{2^{2n-1}}$	$u_1 = \langle -2^{n-1} X_1 \rangle_{2^{2n-1}}$	$u_1 = \langle -2^{n-2} X_1 \rangle_{2^{2n-1}}$
$u_2 = \langle (2^{2n-1} + 2^{n-1}) X_2 \rangle_{2^{2n-1}}$	$u_2 = \langle (2^{2n-2} + 2^{n-2}) X_2 \rangle_{2^{2n-1}}$	$u_2 = \langle (2^{2n-3} + 2^{n-3}) X_2 \rangle_{2^{2n-1}}$
$u_3 = \langle (-2^{2n-1} + 2^{n-1}) X_3 \rangle_{2^{2n-1}}$	$u_3 = \langle (-2^{2n-2} + 2^{n-2}) X_3 \rangle_{2^{2n-1}}$	$u_3 = \langle (-2^{2n-3} + 2^{n-3}) X_3 \rangle_{2^{2n-1}}$

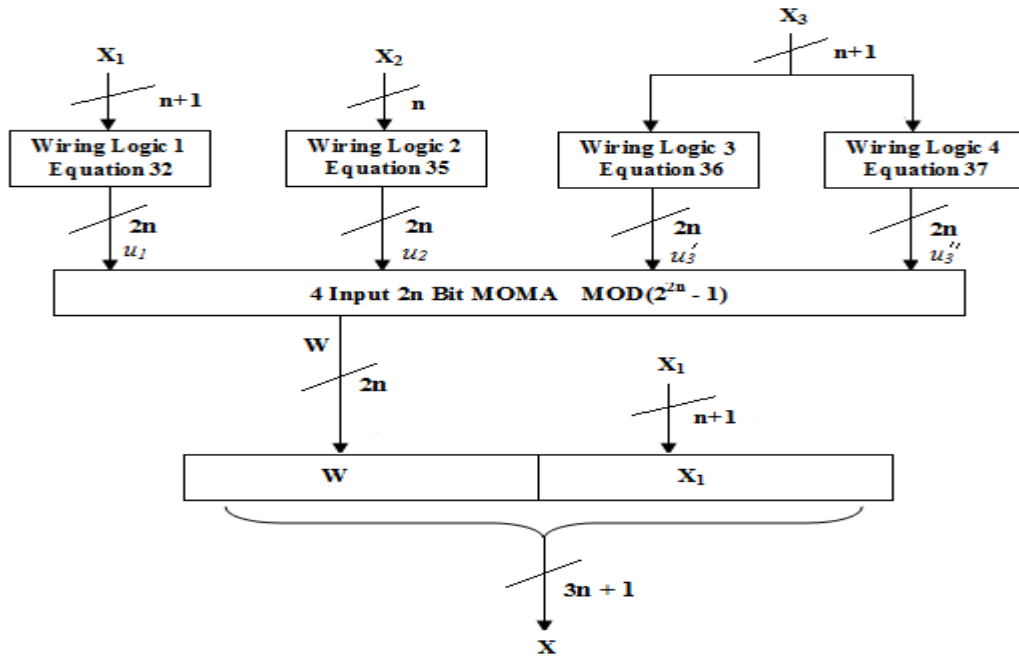


Fig. 6: Implementation of 3 moduli set for  $M_1$

diagram is shown below in figure 5, where  $W = \left\lfloor \frac{X}{2^{n+1}} \right\rfloor$ . The computation of  $X$  is achieved by wiring operations, eliminating the need for multiplication and addition operations.

On similar lines we can derive expressions for  $u_1, u_2$  and  $u_3$  for  $|N|_3=0$  and  $|N|_3=2$ . Table 4 shows  $u_1, u_2$  and  $u_3$  for  $|N|_3=0, |N|_3=1$  and  $|N|_3=2$ .

The following example elaborates the case for  $N = 10; i = |10|_3 = 1$  and  $n = 3$ .

**Example 4.1:** For the Modulus set (16, 7, 9), convert the residue representation (4, 5, 1) into its binary equivalent.

From equation (31) we need to calculate  $u_1, u_2$  and  $u_3$ .  $X_1 = 4; X_1 = X_{13}X_{12}X_{11}X_{10} = 0\ 1\ 0\ 0$ , thus from equation (32), the value of  $u_1$  is given as:

$$u_1 = \bar{X}_{13}\bar{X}_{12}\bar{X}_{11}\bar{X}_{10} | 1\ 1 = 1\ 0\ 1\ 1\ 1\ 1 = 47$$

$X_2 = 5; X_2 = X_{22}X_{21}X_{20} = 1\ 0\ 1$ , from equation (35), the value of  $u_2$  is given as:

$$u_2 = X_{21}X_{20}X_{22} | X_{21}X_{20}X_{22} = 0\ 1\ 1\ 0\ 1\ 1 = 27$$

$X_3 = 1; X_3 = X_{33}X_{32}X_{31}X_{30} = 0\ 0\ 0\ 1$ , from equation (36), (37), the value of  $u_3$  and  $u_3'$  is:



Table 5: Binary form values of  $U_1$ ,  $U_2$  and  $U_3$  for  $|N|_3=0$ ,  $|N|_3=1$  and  $|N|_3=2$

Case	$ N _3=0$	$ N _3=1$	$ N _3=2$
<b>Notation</b>			
<b>For <math>U_1</math></b>	$X_1$ is n bit $X_1 = x_{1(n-1)}x_{1(n-2)} \dots x_{11}x_{10}$ $U_1 = \bar{x}_{1(n-1)}\bar{x}_{1(n-2)} \dots \bar{x}_{11}\bar{x}_{10}$   1 1 ... 1 1	$X_1$ is n + 1 bit $X_1 = x_{1n}x_{1(n-1)} \dots x_{11}x_{10}$ $U_1 = \bar{x}_{1n}\bar{x}_{1(n-1)} \dots \bar{x}_{11}\bar{x}_{10}$   1 1 ... 1 1	$X_1$ is n + 2 bit $X_1 = x_{1(n+1)}x_{1n} \dots x_{11}x_{10}$ $U_1 = \bar{x}_{1(n+1)}\bar{x}_{1n} \dots \bar{x}_{11}\bar{x}_{10}$   1 1 ... 1 1
<b>For <math>U_2</math></b>	$X_2$ is n bit $X_2 = x_{2(n-1)}x_{2(n-2)} \dots x_{21}x_{20}$ $U_2 = x_{20}x_{2(n-1)}x_{2(n-2)} \dots x_{21}$   $x_{20}x_{2(n-1)}x_{2(n-2)} \dots x_{21}$	$X_2$ is n bit $X_2 = x_{2(n-1)}x_{2(n-2)} \dots x_{21}x_{20}$ $U_2 = x_{21}x_{20}x_{2(n-1)}x_{2(n-2)} \dots$   $x_{21}x_{20}x_{2(n-1)}x_{2(n-2)} \dots$	$X_2$ is n bit $X_2 = x_{2(n-1)} \dots x_{22}x_{21}x_{20}$ $U_2 = x_{22}x_{21}x_{20}x_{2(n-1)} \dots$   $x_{22}x_{21}x_{20}x_{2(n-1)} \dots$
<b>For <math>U_3</math></b>	$X_3$ is n + 1 bit $X_3 = x_{3n}x_{3(n-1)} \dots x_{31}x_{30}$ $U_3 = \bar{x}_{30}11 \dots 1$   $1 \bar{x}_{3n}\bar{x}_{3(n-1)} \dots \bar{x}_{31}$ $U_3'' = x_{3n}x_{3(n-1)} \dots x_{31}x_{30}$ 0 0 ... 0 0	$X_3$ is n + 1 bit $X_3 = x_{3n}x_{3(n-1)} \dots x_{31}x_{30}$ $U_3 = \bar{x}_{31}\bar{x}_{30}11 \dots$   $11 \bar{x}_{3n}\bar{x}_{3(n-1)} \dots$ $U_3'' = 0 x_{3n} x_{3(n-1)} \dots   x_{32}x_{31}$ $x_{30} 0 0 \dots 0$	$X_3$ is n + 1 bit $X_3 = x_{3n} \dots x_{32}x_{31}x_{30}$ $U_3 = \bar{x}_{32}\bar{x}_{31}\bar{x}_{30}1 \dots$   $111 \bar{x}_{3n} \dots$ $U_3'' = 0 0 x_{3n} \dots  $ $x_{33}x_{32}x_{31}x_{30}0 \dots 0$

Table 6: Time Delay and Hardware Complexity Comparison

Converter	Moduli Set*	FA	HA	AND/OR	XOR/NOR	Other	Delay
[19]	$M_3$	$6n - 1$	$3n - 7$	----	----	(n-1)MUX	$(4n)t_{FA}$
[15]	$M_0$	$6n + 1$	----	$n + 3$	$n + 1$	2n MUX	$(2n+2)t_{FA} + t_{MUX}$
[17,18]	$M_0$	$4n$	----	2	----	----	$(4n + 1)t_{FA}$
[16] CI	$M_0$	$4n$	1	----	1	2 MUX	$(4n + 1)t_{FA}$
[16] CII	$M_0$	$6n$	1	1	1	(2n+2) MUX	$(n + 1)t_{FA}$
[16] CIII	$M_0$	$4n$	1	$2n + 2$	$2n - 1$	(2n+2) MUX	$(n + 1)t_{FA}$
[14] CI	$M_4$	$4n + 3$	----	N	n	----	$(6n + 5)t_{FA}$
[14] CII	$M_4$	$14n + 21$	$2n + 3$	----	----	(2n+1)3:1MUX	$(2n + 7)t_{FA}$
[14] CIII	$M_4$	$12n + 19$	$2n + 2$	----	----	10(2n+1)A <sub>ROM</sub> (2n+2)2:1MUX	$(2n + 7)t_{FA}$
Ours	$M_0, M_1, M_2$	$6n$	----	----	----	$2n + 1 + i$ Inverters	$(2n+2)t_{FA}$



\*  $M_0 = \{2^n - 1, 2^n, 2^n + 1\}$ ,  $M_1 = \{2^n - 1, 2^{n+1}, 2^n + 1\}$ ,  $M_2 = \{2^n - 1, 2^{n+2}, 2^n + 1\}$ ,  $M_3 = \{2^n - 1, 2^n, 2^{n-1} - 1\}$  and  $M_4 = \{2^{n+1} - 1, 2^n, 2^n - 1\}$ .

Table 7: Comparison of various Converters for Three Moduli set in terms of Dynamic Range Efficiency (Deff)

Word Length (N)	Value of n	Moduli Set	$D_{eff} = M/2^N$
$ N _3 = 0$	N = 3n	$M_0$	99%
		$M_1$	200%
		$M_2$	393%
		$M_3$	33%
		$M_4$	164%
$ N _3 = 1$	N = 3n + 1	$M_0$	50%
		$M_1$	99%
		$M_2$	197%
		$M_3$	17%
		$M_4$	82%
$ N _3 = 2$	N = 3n + 2	$M_0$	25%
		$M_1$	50%
		$M_2$	99%
		$M_3$	8%
		$M_4$	41%

$$u_3 = \overline{X}_{31}\overline{X}_{30} | 11 \overline{X}_{33}\overline{X}_{32} = 101111$$

$$\text{And } u_3'' = 0 X_{33} | X_{32}X_{31}X_{30}0 = 000010$$

Again, from the assumption that  $\left\lfloor \frac{x}{2^{n+1}} \right\rfloor = W$  and following equation (31),

$$\left\lfloor \frac{x}{2^{n+1}} \right\rfloor = W = \langle u_1 + u_2 + u_3 \rangle_{2^{2n-1}}$$

$$W = \langle 47 + 27 + 49 \rangle_{63} = 111100 = 60 \quad \begin{matrix} W & X_1 \end{matrix}$$

$$\text{From equation (30) } X = W * 2^{n+1} + X_1 = 11110000100 = 964 \quad (39)$$

Here we can see that X is obtained by shifting W, n + 1 bit to the left and appending  $X_1$ . Figure 6 shows the implementation of this example.

**Theorem 3:** For any word length N, the three moduli set is  $\{2^{n+i}, 2^n - 1, 2^n + 1\} = \{m_1, m_2, m_3\}$ , where  $i = |N|_3$ . Any integer X in  $[0, M - 1]$  has RNS representation of  $(x_1, x_2, x_3)$ , where  $X_j = |X|_{m_j}$  and the residue size is  $(n + i, n, n + 1)$  respectively. The Value of X can be computed from  $X = W * 2^{n+i} + X_1$

Where  $W = \langle u_1 + u_2 + u_3 \rangle_{2^{2n-1}}$  and  $u_j = f(x_j)$  in 2n bit representation. The value of  $u_j$  is summarized in table 5 for each i. The size of X is 3n + i.

**Hardware Architecture:**

Theorem 3 states that regardless of i, the hardware architecture is the same. It consists of rewiring each residue  $x_j$  to compute  $u_j$  in 2n bit representation. The  $u_j$ 's are then added mod  $(2^{2n} - 1)$  to compute W. The final answer X is computed by shifting W by n + i bit and appending  $X_1$  of size n + i bit, i.e. no overlapped. The hardware architecture is shown in figure 5 regardless of i. The only hardware to be implemented is the mod  $(2^{2n} - 1)$  adder, which is shown in figure 7. The propagation delay and hardware count are determined by the mod  $(2^{2n} - 1)$  adder. The wiring operations only require inverters and introduce one inverter delay. The number of inverters required is determined when computing  $u_j$ .  $u_1$  needs to compute  $-X_1$ , which requires n + 1 (size of  $X_1$ ) inverters;  $u_2$  does not require any inverter and  $u_3$  needs to compute  $-X_3$  which requires n + 1 (size of  $X_3$ ) inverters i.e. total number of inverters is n + 1 + i, where  $i = |N|_3$ .

The mod adder architecture consists of 2 CSA (2n) and each CSA (2n) contribute 1 FA delay. The 2 operands mod  $(2^{2n} - 1)$  adder is implemented using RCA (2n) which contributes 2n

FA delay. The 1 inverter delay is insignificant compared to FA delay and thus the total delay is  $2n + 2$ . Each CSA ( $2n$ ) has  $2n$

FA and RCA ( $2n$ ) has  $2n$  FA, therefore a total of  $6n$  FAs are used. The mod adder architecture is shown in figure 7.

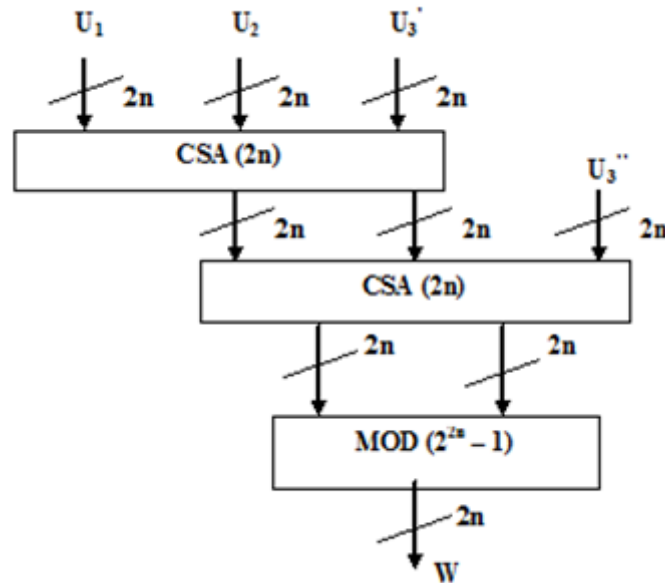


Fig. 7: Architecture of mod  $2^{2n} - 1$  adder

#### V. COMPARISION

In table 6, the comparison is done in terms of delay time and table 7 did the comparison in terms of dynamic range [20-23]. Looking at table 6 we can say that in our case we are using only  $6n$  full adders and  $2n + 1 + i$  inverters, whereas all other converters require the use of one or more components such as gates (OR, NAND, XOR, NOR), Half Adders and Multiplexers in addition to Full adders, thus reducing the hardware complexity to its minimum threshold value. If we look at the delay time, we are superior to all the converters except CII, CIII [16].

In table 7, we compare the Dynamic Range Efficiency ( $D_{eff}$ ) of the 5 Moduli sets with respect to word length  $N$ . The best  $D_{eff}$  of each moduli set is as follows:

$M_0$	$D_{eff} = 99\%$ if $ N _3 = 0$
$M_1$	$D_{eff} = 99\%$ if $ N _3 = 1$
$M_2$	$D_{eff} = 99\%$ if $ N _3 = 2$
$M_3$	$D_{eff} = 33\%$ if $ N _3 = 0$
$M_4$	$D_{eff} = 82\%$ if $ N _3 = 1$

Thus, we can say that our proposed moduli set  $M_0$ ,  $M_1$  and  $M_2$  are the most efficient as far as  $D_{eff}$  is concerned.

The proposed Moduli set  $M_0$ ,  $M_1$  and  $M_2$  have the following properties.

1. Excellent Propagation Delay.
2. Simple Hardware Architecture.
3. The Best  $D_{eff}$ .

#### VI. CONCLUSION

In this paper we proposed a new three moduli set  $(2^n - 1, 2^{n+i}, 2^n + 1)$ , where  $i = |N|_3$ . It is shown to have the best Dynamic Range Efficiency for any  $N$ . The CRT implementation has the same architecture for any  $i$ . It consists of wiring operations and one mod adder operations. The Hardware complexity is  $6n$  FA,  $2n + 1 + i$  inverters with competitive propagation delay of  $(2n + 2)t_{FA}$ .

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