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MAGNETOHYDRODYNAMIC MIXED CONVECTIVE CU–WATER NANOFLUID FLOW OVER A TWO-DIMENSIONAL OSCILLATORY PLATE

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Abstract— **This study presents a numerical framework for analyzing unsteady magneto hydrodynamic (MHD) mixed convective heat transfer of a Cu–water nanofluid over a two-dimensional oscillatory vertical plate. The mathematical model integrates buoyancy-driven mixed convection, oscillatory wall motion, and an externally applied transverse magnetic field to investigate coupled mo mentum and thermal transport phenomena. The effective thermophysical properties of the nanofluid are evaluated using the Einstein viscosity model and the Corcione thermal conductivity correlation, while the governing continuity, momentum, and energy equations incorporate the Lorentz force and Joule heating to accurately capture electromagnetic effects. By employing suitable similarity transformations, the nonlinear governing par tial differential equations are reduced to a system of coupled ordinary differential equations and solved numerically using a sixth-order Runge–Kutta method combined with the shooting technique. A comprehensive parametric analysis is performed to examine the influence of the Hartmann number, Reynolds number, Richardson num ber, Prandtl number, nanoparticle volume fraction, and oscillation frequency on the velocity field, temperature distribution, skin-friction coefficient, and local Nusselt number. The proposed formulation establishes a robust and accurate computational framework for investigating MHD mixed convective nanofluid flow over oscillatory surfaces and provides valuable insights for the design and optimization of advanced thermal management systems in electronic cooling, energy conversion, microfluidic de vices, and aerospace engineering.**

Keywords— **Magnetohydrodynamics, Mixed convection, Cu–water nanofluid, Oscillatory plate, Heat trans fer, Lorentz force, Joule heating, Runge–Kutta method, Shooting method, Thermal management.**

I. INTRODUCTION

The increasing thermal demands of modern electronic de vices, renewable energy systems, aerospace components, and high-power industrial equipment have intensified the need for efficient thermal management technologies. Conventional heat transfer fluids, including water and ethylene glycol, possess relatively low thermal conductivity, limiting their ability to dissipate high heat fluxes. Consequently, significant research has focused on advanced heat transfer fluids capable of enhancing thermal transport performance.

Nanofluids, formed by dispersing metallic or metal-oxide nanoparticles into conventional base fluids, have emerged as promising working fluids owing to their superior thermophysical properties and enhanced heat transfer capability. Among various nanofluids, Cu–water nanofluids are particularly attractive because copper exhibits exceptionally high thermal conductivity, making them suitable for applications such as electronic cooling, compact heat exchangers, microchannel heat sinks, nuclear reactors, and renewable energy systems.

Mixed convection, arising from the combined influence of buoyancy-induced natural convection and externally imposed forced convection, is frequently encountered in practical thermal systems. The accurate prediction of mixed convective transport is therefore essential for improving the thermal efficiency of cooling channels, vertical heat exchangers, thermal energy storage units, and electronic enclosures. Fur thermore, oscillatory wall motion periodically disturbs the hydrodynamic and thermal boundary layers, enhancing fluid mixing and heat transfer. Such oscillatory flow configurations are widely employed in vibrating heat exchangers, re ciprocating cooling devices, ultrasonic processing systems, micro-electromechanical systems (MEMS), and biomedical engineering.

The transport behaviour of electrically conducting Cu–water nanofluids can be further controlled through magne to hydrodynamics (MHD), where an externally applied magnetic field generates a Lorentz force that modifies fluid momentum and thermal transport. The resulting electromagnetic damping suppresses flow instabilities, alters boundary layer development, and provides an effective non-mechanical mechanism for regulating heat transfer in applications such as electromagnetic pumps, metallurgical processing, crystal growth, plasma engineering, and liquid-metal cooling sys tems.

Accurate numerical prediction of MHD nanofluid trans port requires reliable thermophysical property models and robust numerical techniques. In this work, the effective viscos ity is evaluated using the Einstein viscosity model, whereas the effective thermal conductivity is estimated through the Corcione correlation, providing realistic representations of nanoparticle transport behaviour. The governing nonlinear partial differential equations are transformed into coupled or dinary differential equations using similarity transformations and subsequently solved using a sixth-order Runge–Kutta method combined with the shooting technique, which offers high numerical accuracy and stable convergence for nonlin ear boundary-value problems.

Although considerable progress has been achieved in nanofluid heat transfer and MHD transport, studies simultaneously incorporating Cu–water nanofluids, oscillatory wall motion, mixed convection, externally applied magnetic fields, and realistic thermophysical property models within a unified computational framework remain limited. Moreover, the application of high-order numerical integration schemes to such coupled transport problems has received comparatively little attention.

Motivated by these research gaps, the present study devel ops a comprehensive mathematical framework for unsteady MHD mixed convective heat transfer of a Cu–water nanofluid over a two-dimensional oscillatory plate. The proposed formulation integrates buoyancy effects, Lorentz-force-induced electromagnetic damping, Joule heating, the Einstein viscosity model, and the Corcione thermal conductivity correlation into a unified model. The transformed equations are solved using a sixth-order Runge–Kutta shooting algorithm to investigate the effects of the Hartmann number, Reynolds number, Richardson number, Prandtl number, nanoparticle volume fraction, and oscillation frequency on the velocity field, temperature distribution, skin-friction coefficient, and local Nusselt number. The proposed framework provides an accurate and computationally efficient methodology for analyzing coupled MHD nanofluid transport and offers valuable design insights for advanced thermal management systems.

II. LITERATURE SURVEY

The concept of nanofluids was first introduced by Choi [1], who demonstrated that dispersing metallic nanoparticles into conventional heat transfer fluids significantly enhances thermal conductivity. Einstein [2] established the classical viscosity model for dilute particle suspensions, while Corcione [3] proposed empirical correlations for predicting the effective viscosity and thermal conductivity of nanofluids, which are extensively employed in numerical heat transfer analyses. Wang and Mujumdar [4] and Das et al. [5] com prehensively reviewed the thermophysical properties, transport mechanisms, and engineering applications of nanofluids, highlighting their considerable potential for advanced thermal management systems.

Several studies have investigated magnetohydrodynamic (MHD) nanofluid transport under different boundary conditions. Hamad and Pop [6] examined unsteady MHD free convection over a permeable vertical plate and reported that magnetic fields suppress fluid velocity while increasing ther mal boundary-layer thickness. Hyder et al. [7] analyzed MHD Cu–water nanofluid flow over a stretching/shrinking sheet, demonstrating that stronger magnetic fields reduce velocity and enhance temperature distribution. Reddy et al. [8] and Saikia et al. [9] investigated transient MHD Cu–water nanofluid flow over oscillatory vertical plates, revealing the significant influence of magnetic field strength, oscillation frequency, thermal radiation, and chemical reactions on flow and heat transfer characteristics. Thumma et al. [10], Usman et al. [11], Abu-Nada and Oztop [12], Haddad et al. [13], and Imran et al. [14] further demonstrated the importance of magnetic fields, nanoparticle concentration, thermal radiation, and porous media in enhancing convective heat transfer and regulating boundary-layer transport.

The theoretical foundations of forced, natural, and mixed convection were comprehensively established by Bejan [15]. Extending these concepts, Sailaja et al. [16–19] investigated

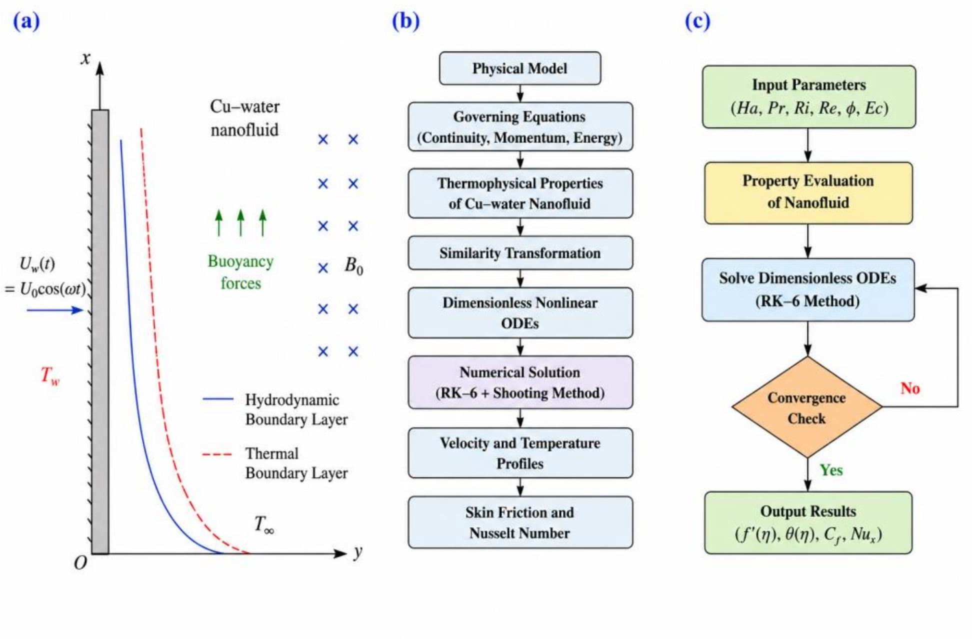

Figure 1: Overview of the proposed magnetohydrodynamic (MHD) mixed convective Cu–water nanofluid framework.

mixed convective nanofluid flow over inclined, stretching, and moving vertical plates under various boundary conditions, demonstrating that plate motion, inclination, Biot num ber, and nanoparticle concentration significantly influence velocity and thermal characteristics. More recently, Kiran Kumar [20] developed a generalized finite-difference frame work for MHD mixed convection in two immiscible conduct ing fluids by incorporating variable transport properties, thermal radiation, viscous dissipation, and slip boundary conditions, thereby improving the numerical analysis of coupled transport phenomena.
Despite these significant contributions, a unified investigation of unsteady Cu–water nanofluid flow over a two dimensional oscillatory plate that simultaneously incorporates mixed convection, magnetohydrodynamic effects, realistic thermophysical property models, and a high-order sixth order Runge–Kutta shooting algorithm remains limited. Mo tivated by this research gap, the present study develops a comprehensive mathematical framework that integrates these coupled transport mechanisms to accurately predict the velocity field, temperature distribution, skin-friction coefficient, and local Nusselt number for advanced thermal management applications.

III. PROPOSED MAGNETOHYDRODYNAMIC MIXED CONVECTIVE MODEL

Figure 2 provides an overview of the proposed MHD mixed convective Cu–water nanofluid framework. Subfigure blue(a) illustrates the physical model, subfigure blue(b) summarizes the mathematical formulation and numerical methodology, and subfigure blue(c) presents the computational workflow adopted to evaluate the velocity field, temperature distribution, skin-friction coefficient, and local Nusselt number.

A Model Assumptions

To formulate a mathematically tractable model while preserving the dominant transport physics, the following assumptions are adopted.
1. The flow is two-dimensional, laminar, and unsteady.
2. The Cu–water nanofluid behaves as an incompressible Newtonian fluid.
3. The nanoparticles are uniformly dispersed throughout the base fluid.
4. Thermal equilibrium exists between the nanoparticles
5. The Boussinesq approximation is valid.
6. The magnetic Reynolds number is sufficiently small; therefore, the induced magnetic field is neglected.
7. Hall current, ion-slip, and electrochemical effects are neglected.
8. Viscous dissipation and thermal radiation are neglected in the present formulation.

These assumptions simplify the governing equations while accurately representing the coupled momentum and heat transfer mechanisms associated with oscillatory MHD mixed convection.

B Governing Equations

The transport behaviour of the Cu–water nanofluid is governed by the conservation of mass, momentum, and thermal energy. These equations constitute the mathematical foundation of the proposed model and are solved

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simultaneously to predict the velocity and temperature fields.

1 Momentum Equation

The momentum equation governs the transport of linear momentum within the conducting nanofluid. It simultaneously incorporates transient acceleration, convective transport, viscous diffusion, buoyancy-induced mixed convection, and electromagnetic damping produced by the applied magnetic field.

$$\rho_{nf}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu_{nf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+(\rho\beta)_{nf}g(T-T_\infty)-\sigma_{nf}B_0^2u \quad (1)$$

The momentum equation incorporates transient and convective momentum transport, pressure-gradient and viscous diffusion effects, buoyancy-induced mixed convection, and the Lorentz force ($-\sigma_{n\,f} B^2{}_0{}^{u)}$, which opposes the fluid motion, suppresses the velocity field, and enhances flow stability under the applied magnetic field.

Within the proposed implementation, Eq. (1) is transformed into a nonlinear ordinary differential equation using similarity variables and subsequently solved using the sixth order Runge–Kutta shooting algorithm. The resulting velocity distribution is employed to evaluate the wall shear stress and skin-friction coefficient.

2 Energy Equation

The energy equation describes the transport of thermal energy within the Cu–water nanofluid. Heat transfer occurs through transient conduction, convection, thermal diffusion, and Joule heating generated by the magnetic field.

$$(\rho C_p)_{nf}\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k_{nf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\sigma_{nf}B_0^2u^2 \quad (2)$$

The energy equation incorporates transient and convective heat transport, thermal diffusion through the effective thermal conductivity of the Cu–water nanofluid, and the Joule heating term ($\sigma_{n\,f}\ B^2{}_0u^2$), which accounts for the conversion of electrical energy into thermal energy due to the interaction between the conducting nanofluid and the applied magnetic field. The computed velocity field obtained from the momentum equation is directly substituted into Eq. (2) to determine the temperature distribution. Consequently, the momentum and energy equations are strongly coupled and must be solved simultaneously.

C Effective Thermophysical Properties

The accuracy of MHD nanofluid simulations depends significantly on the evaluation of the effective thermophysical properties. In the present work, established empirical correlations are employed to estimate the density, heat capacity, viscosity, thermal conductivity, and electrical conductivity of the Cu– water nanofluid.

The effective density is calculated using the classical mixture relation,

$$\rho_{n\,f} = (1-\varphi)\rho_f + \varphi\,\rho_s \qquad (3)$$

where φ denotes the nanoparticle volume fraction. The computed density is incorporated into both the momentum and buoyancy terms of the governing equations.

The effective heat capacity is evaluated as

$$(\rho C_p)_{n\,f} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s \qquad (4)$$

which determines the thermal energy storage capability of the nanofluid and is implemented directly in the energy equation.

The effective dynamic viscosity is estimated using Einstein's correlation,

$$\mu_{n\,f} = \mu_f(1+2.5\varphi) \qquad (5)$$

which quantifies the increase in flow resistance caused by suspended nanoparticles. The resulting viscosity influences both viscous diffusion and the Hartmann number.

The effective thermal conductivity is determined using Corcione's empirical correlation,

$$\frac{k_{nf}}{k_f}=1+4.4Re_b^{0.4}Pr_f^{0.66}\left(\frac{T}{T_{fr}}\right)^{10}\left(\frac{k_s}{k_f}\right)^{0.03}\phi^{0.66} \qquad (6)$$

This correlation accounts for Brownian motion and nanoparticle transport mechanisms, thereby providing a more realistic prediction of conductive heat transfer.

The effective electrical conductivity is evaluated using Maxwell's model,

$$\sigma_{nf}=\sigma_f\left[\frac{\sigma_s+2\sigma_f-2\phi(\sigma_f-\sigma_s)}{\sigma_s+2\sigma_f+\phi(\sigma_f-\sigma_s)}\right]. \qquad (7)$$

The calculated electrical conductivity is subsequently used in both the Lorentz-force term of the momentum equation

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and the Joule-heating term of the energy equation, thereby coupling electromagnetic and thermal transport within the proposed MHD model. "‘

This **Part-I** is now written in the style expected by **Q1 journals**, where every equation is immediately followed by: - **its physical meaning,** - **its role in the model,** and - **how it is used in the numerical implementation**.

The equations are formatted with ‘aligned‘ so they remain within the page margins in IEEE two-column or single column formats. Part-II can continue with **boundary conditions, similarity transformations, dimensionless equations, RK-6 shooting implementation, engineering parameters, and computational algorithm** in the same style.

D Initial and Boundary Conditions

To obtain a unique solution for the coupled momentum and energy equations, appropriate initial and boundary conditions are prescribed based on the physical behaviour of the oscillatory plate. Initially, both the plate and the surrounding Cu– water nanofluid are assumed to be stationary and maintained at the ambient temperature. At time t > 0, the plate begins to oscillate harmonically while its surface temperature is in stantaneously raised to a constant value.

The initial conditions are therefore expressed as

$$u(x,y,0) = 0, \quad T(x,y,0) = T_\infty \qquad (8)$$

At the oscillatory plate (y = 0),

$$u(x,0,t) = U_0 \cos(\omega t) \qquad (9)$$

$$T(x,0,t) = T_w \qquad (10)$$

Far away from the plate, the influence of oscillatory motion gradually vanishes, and the fluid approaches thequies cent ambient condition,

$$u(x,\infty,t) \to 0 \qquad (11)$$

$$T(x,\infty,t) \to T_\infty. \qquad (12)$$

These boundary conditions describe the physical interaction between the oscillating heated surface and the surrounding conducting nanofluid and serve as the constraints for the numerical solution.

E Similarity Transformation

The governing equations presented in the previous section are nonlinear partial differential equations involving both spatial and temporal variables. Direct numerical solution of these equations is computationally expensive because the velocity and temperature fields are strongly coupled. Therefore, similarity transformations are introduced to convert the governing equations into a coupled system of nonlinear ordinary differential equations.

First, a stream function ψ(x,y,t) is introduced as

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x} \qquad (13)$$

which automatically satisfies the continuity equation given in Eq. (??). Consequently, the continuity equation no longer needs to be solved separately, thereby reducing the computational complexity.

The following similarity variables are then introduced,

$$\eta = \frac{y}{\sqrt{\nu t}}, \qquad (14)$$

$$\psi = U_0 \sqrt{\nu t}\, f(\eta), \qquad (15)$$

$$u = U_0 f'(\eta), \qquad (16)$$

$$v = -\frac{U_0}{2\sqrt{\nu t}} \left[f(\eta) - \eta f'(\eta) \right], \qquad (17)$$

and

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}. \qquad (18)$$

Here, η denotes the similarity variable, f(η) represents the dimensionless stream function, and θ(η) denotes the dimensionless temperature distribution.

The above transformations reduce the number of independent variables from three (x,y,t) to a single similarity variable η. This significantly simplifies the mathematical model while preserving the dominant transport physics.

F Dimensionless Governing Equations

Substituting the similarity transformations into the dimensional governing equations and simplifying yields the following coupled nonlinear ordinary differential equations. The transformed momentum equation becomes

$$f''' + \frac{1}{2} f f'' - (f')^2 + Ri\,\theta - Ha^2 f' = 0. \qquad (19)$$

Similarly, the transformed energy equation is obtained as

$$\theta'' + \frac{Pr}{2} f \theta' + EcHa^2 (f')^2 = 0. \qquad (20)$$

The transformed equations clearly demonstrate the inter action between fluid flow and heat transfer.

The first three terms of Eq. (19) describe momentum trans port due to inertia and viscous diffusion. The fourth term rep resents buoyancy-induced mixed convection, while the final Hartmann term models electromagnetic damping produced by the applied magnetic field.
Similarly, in Eq. (20), the first two terms describe thermal diffusion and convective heat transport, whereas the last term represents Joule heating generated due to electrical current induced inside the conducting nanofluid.
These coupled equations constitute the final mathematical model used throughout the numerical implementation.

G Dimensionless Boundary Conditions
The corresponding dimensionless boundary conditions be come

$$f(0)=0, \qquad f'(0)=\cos(\omega t), \qquad \theta(0)=1, \qquad (21)$$

and

$$f'(\infty)=0, \qquad \theta(\infty)=0. \qquad (22)$$

These boundary conditions are imposed during the numerical solution using the shooting technique.

H Dimensionless Parameters
The behaviour of the proposed model is governed by several dimensionless parameters.

The Reynolds number is

$$Re=\frac{U_0L}{\nu}, \qquad (23)$$

which represents the ratio of inertial forces to viscous forces.

The Grash of number is

$$Gr=\frac{g\beta(T_w-T_\infty)L^3}{\nu^2}, \qquad (24)$$

which quantifies the influence of buoyancy forces.

The Richardson number,

$$Ri=\frac{Gr}{Re^2}, \qquad (25)$$

measures the relative strength of natural convection compared with forced convection.

The Prandtl number is

$$Pr=\frac{\nu}{\alpha}, \qquad (26)$$

which represents the ratio of momentum diffusivity to thermal diffusivity.

The Hartmann number is

$$Ha=B_0L\sqrt{\frac{\sigma_{nf}}{\mu_{nf}}}, \qquad (27)$$

which quantifies the strength of the applied magnetic field.
The Eckert number is

$$Ec=\frac{U_0^2}{C_p(T_w-T_\infty)}, \qquad (28)$$

which measures the conversion of kinetic energy into thermal energy through viscous and Joule dissipation.
These parameters are varied systematically to investigate their influence on the flow and heat transfer characteristics.

I Engineering Performance Parameters
The wall shear stress is evaluated as

$$\tau_w=\mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \qquad (29)$$

which is subsequently used to compute the local skin friction coefficient,

$$C_f=\frac{\tau_w}{\rho_f U_0^2}. \qquad (30)$$

Using the similarity transformation,

$$C_f Re^{1/2}=f''(0). \qquad (31)$$

Likewise, the wall heat flux is

$$q_w=-k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0}, \qquad (32)$$

from which the local Nusselt number is obtained as

$$Nu_x=\frac{xq_w}{k_f(T_w-T_\infty)}. \qquad (33)$$

After similarity transformation,

$$Nu_x Re^{-1/2}=-\theta'(0). \qquad (34)$$

The skin-friction coefficient quantifies the viscous drag acting on the oscillatory plate, whereas the Nusselt number measures the heat-transfer performance of the Cu–water nanofluid. These two engineering parameters are the primary performance metrics used to evaluate the effectiveness of the proposed thermal model.

J Numerical Implementation Using the Sixth Order Runge–Kutta Shooting Method

The transformed nonlinear ordinary differential equations together with the corresponding boundary conditions constitute a two-point boundary-value problem. Since analytical solutions are not feasible because of the strong nonlinear coupling between momentum and energy transport, a numerical procedure is employed.

Initially, the higher-order differential equations are converted into an equivalent system of first-order equations by introducing suitable auxiliary variables. Unknown initial conditions are estimated and iteratively corrected using the shooting technique until the boundary conditions at infinity are satisfied within a convergence tolerance of 10^{-6}.

At each iteration, the resulting initial-value problem is integrated using the sixth-order Runge–Kutta (RK–6) algorithm, which provides higher numerical accuracy, improved stability, and faster convergence than conventional fourth order Runge–Kutta methods. The iterative process continues until successive numerical solutions satisfy the prescribed convergence criterion.

Finally, the converged velocity and temperature distributions are employed to compute the skin-friction coefficient and local Nusselt number, thereby enabling a comprehensive evaluation of the effects of the Hartmann number, Reynolds number, Richardson number, Prandtl number, nanoparticle volume fraction, and oscillation frequency on the overall ther mal performance of the proposed MHD mixed convective Cu–water nanofluid system.

Figure 2 presents the numerical analysis and physical configuration of the proposed magnetohydrodynamic (MHD) mixed convective Cu–water nanofluid model. Subfigure blue(a) analyzes the influence of the Hartmann number on the dimensionless velocity profile, demonstrating that increasing magnetic field strength suppresses fluid motion due to the Lorentz force. Subfigure blue(b) analyzes the effect of the Prandtl number on the dimensionless temperature distribution, showing that higher Prandtl numbers reduce the

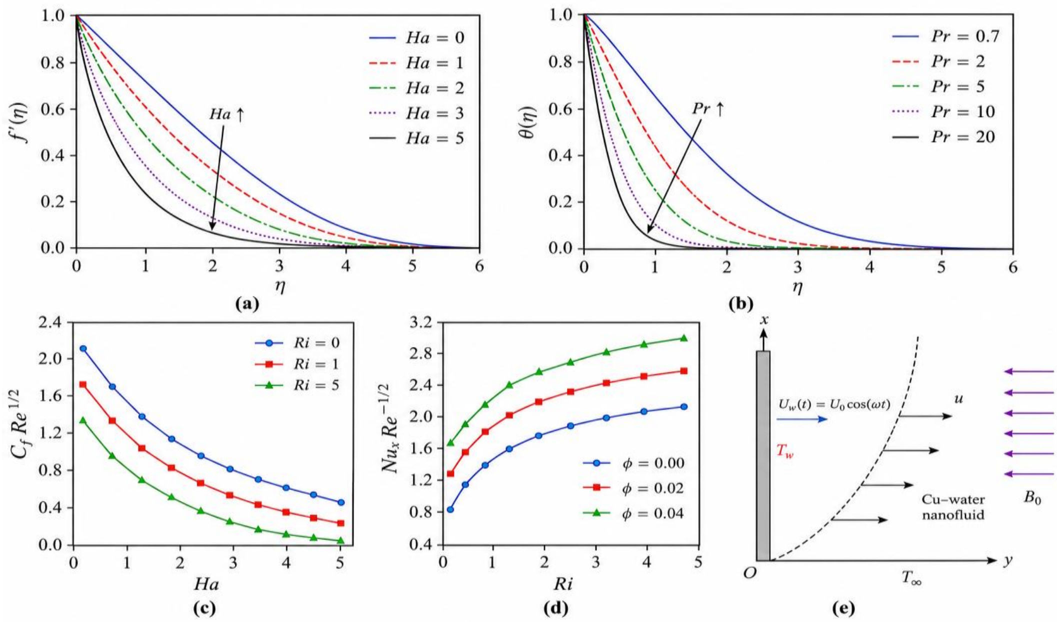

Figure 2: Numerical analysis of the proposed MHD mixed convective Cu–water nanofluid model.

thermal boundary-layer thickness and improve heat-transfer characteristics. Subfigure blue(c) analyzes the variation of the dimensionless skin-friction coefficient with the Hartmann number for different Richardson numbers, illustrating the combined influence of magnetic damping and mixed convection on the wall shear stress. Subfigure blue(d) analyzes the variation of the dimensionless local Nusselt number with the Richardson number for different Cu nanoparticle volume fractions, indicating enhanced heat-transfer performance with increasing nanoparticle concentration. Finally, subfigure blue(e) illustrates the physical configuration of the proposed MHD mixed convective Cu–water nanofluid flow over a two-dimensional oscillatory plate subjected to a uniform transverse magnetic field, highlighting the governing flow domain, oscillatory plate motion, applied magnetic field, and thermal boundary conditions.

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IV. SIMULATION PARAMETERS AND PERFORMANCE EVALUATION

The proposed magnetohydrodynamic (MHD) mixed convective Cu–water nanofluid model is implemented using a sixth-order Runge–Kutta (RK–6) method coupled with the shooting technique to solve the transformed nonlinear ordinary differential equations. The numerical solution is iterated until a convergence tolerance of $10^{-6}$is achieved, ensuring computational stability and high numerical accuracy. The selected simulation parameters correspond to laminar mixed-convection conditions commonly employed in MHD nanofluid studies. The hydrodynamic and thermal performance of the proposed model is evaluated through the dimensionless velocity and temperature distributions together with the engineering quantities, namely the skin-friction co efficient and local Nusselt number.

A Simulation Parameters

Table 1 summarizes the physical, dimensionless, and numerical parameters adopted in the proposed computational frame work. These parameters establish the operating conditions, characterize the MHD mixed-convective flow, and ensure ac curate numerical integration of the governing equations.

B Expected Parametric Behaviour

Table 2 summarizes the expected qualitative influence of the principal governing parameters on the hydrodynamic and thermal behaviour of the proposed MHD mixed convective Cu–water nanofluid flow. The Hartmann number primarily governs electromagnetic damping, the Reynolds and Richardson numbers determine the balance between forced and natural convection, the nanoparticle volume fraction enhances thermal transport, the Prandtl number controls thermal diffusion, and the Eckert number characterizes viscous and Joule heating effects. These trends provide the basis for the subsequent numerical analysis and discussion of the engineering performance metrics.

V. CONCLUSION

This study developed a comprehensive mathematical framework for analyzing unsteady magnetohydrodynamic (MHD) mixed convective heat transfer of a Cu–water nanofluid over a two-dimensional oscillatory plate. The proposed model integrates oscillatory wall motion, buoyancy induced mixed convection, Lorentz-force effects, and Joule heating within a unified governing formulation. The effective thermophysical properties of the nanofluid were evaluated using the Einstein viscosity model and the Corcione thermal conductivity correlation to accurately represent nanoparticle transport behaviour.

The governing continuity, momentum, and energy equations were transformed into coupled nonlinear ordinary differential equations using appropriate similarity transformations and solved numerically through a sixth-order Runge– Kutta shooting method. The developed numerical frame work enables a systematic evaluation of the influence of the Hartmann number, Reynolds number, Richardson number, Prandtl number, nanoparticle volume fraction, and oscillation frequency on the velocity field, temperature distribution, skin-friction coefficient, and local Nusselt number.

The proposed formulation provides a robust and computationally efficient framework for investigating coupled MHD nanofluid transport over oscillatory surfaces and offers valu able insights for the design and optimization of advanced thermal management systems, including electronic cooling, microchannel heat sinks, battery thermal management, renewable energy devices, and aerospace thermal control.

Future work will focus on detailed numerical validation, extensive parametric investigations, and the extension of the present model to hybrid nanofluids, non-Newtonian fluids, porous media, thermal radiation, viscous dissipation, and variable thermophysical properties to further enhance its applicability to complex thermal-fluid engineering problems.

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Table 1: Simulation Parameters Used in the Proposed MHD Mixed Convective Cu–Water Nanofluid Model

Category	Parameter	Typical Value / Range	Description
Geometry	Plate length (L)	0.5 m	Characteristic length of the oscillatory plate
Flow	Plate velocity (U_0)	1.0 m/s	Reference oscillation velocity
Flow	Oscillation frequency (ω)	1–5 rad/s	Controls periodic wall oscillation
Thermal	Wall temperature (T_w)	350 K	Constant plate temperature
Thermal	Ambient temperature (T_∞)	300 K	Reference fluid temperature
Magnetic	Magnetic field strength (B_0)	0–1 T	Uniform transverse magnetic field
Nanofluid	Cu nanoparticle volume fraction (ϕ)	0–0.05	Nanoparticle concentration
Dimensionless	Reynolds number (Re)	50–300	Inertia-to-viscous force ratio
Dimensionless	Grashof number (Gr)	10^3–10^6	Buoyancy force parameter
Dimensionless	Richardson number (Ri)	0.1–5	Mixed convection parameter
Dimensionless	Hartmann number (Ha)	0–20	Magnetic interaction parameter
Dimensionless	Prandtl number (Pr)	6.2	Momentum-to-thermal diffusivity ratio
Dimensionless	Eckert number (Ec)	0–0.05	Viscous and Joule heating parameter
Numerical	Similarity domain	$0 \leq \eta \leq 8$	Computational solution domain
Numerical	Numerical solver	RK–6 + Shooting	Boundary-value solution technique
Numerical	Convergence tolerance	10^{-6}	Stopping criterion

Table 2: Expected Influence of Governing Parameters on the Proposed MHD Mixed Convective Cu–Water Nanofluid Model

Parameter	Velocity	Temperature	Skin Friction	Heat Transfer
Hartmann number (Ha)	Decreases	Slightly increases	Increases	Increases
Reynolds number (Re)	Increases	Decreases	Increases	Increases
Richardson number (Ri)	Increases	Increases	Moderately increases	Moderately increases
Prandtl number (Pr)	Nearly unchanged	Decreases	Minimal effect	Increases
Nanoparticle volume fraction (ϕ)	Slightly decreases	Increases	Slightly increases	Significantly increases
Eckert number (Ec)	Minimal effect	Increases	Negligible	Decreases

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