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ISS STABILITY CONTROL: A COMPARISON OF DIFFERENT APPROACHES

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***Abstract*—The present paper is characterized by the intense element of simulation, is formed in terms of studying the ISS stability, either by definition – primarily – or by comparing - secondarily, to Lyapunov stability and BIBO. The used method will be comprised by a route in the study of modern and additionally classic references of our subject. Then, the references will be followed by a series of simulations in MATLAB, in a known or an unknown manner. The qualitative/topological – in mathematical terms – and the quantitative/statistical analysis of the results will be presented in the final pages, whereas an interdisciplinary role will be attached to the query based research to people in industry or academic cycles, who use or research ISS stability. On the other hand, we shall be given the opportunity to extract – awaited – results, that will be formulated in terms of complex mathematical structures, which are supposed to connect Control Theory of non-linear nature to those of linear one and to the advantages of using ISS stability. Naturally, the epilogue will be the intense use, in terms of theoretical results on the one hand, but on the other how can the method be utilized by the scientifically-oriented Engineer, either in human measures or in production.**

Keywords—Automatic Control Systems, Linear Systems, Stability, Lyapunov Stability, Simulation

Mathematics Subject Classification (2010)—37B25, 37B55, 39A06, 39A30, 49K15, 49K20, 49K40, 93B05, 93B07, 93B18,93B52, 93C05, 93C10, 93C15, 93C20, 93C85, 93D05, 93D09, 93D20, 93D25, 93D30.

I. INTRODUCTION

The present article is about exposing a direct comparison between the – relatively – old Lyapunov stability[1] and the ISS stability. While the main generalization is the passing to the non-linear case from the linear one, one has to determine some other analytical or topological characteristics, too.

A. Some History

Before continuing to the main prerequisites for understanding the article and its corresponding thesis, we should include some historical aspects of this theory. Automatic Control Theory began as a consequence of Control Theory in systems that could be automated by certain factors, while being understood by Ordinary or Partial Differential Equations and Systems of these, Dynamical Systems and Deterministic Chaos incontinuous, or Difference Equations in discrete problems, whilst additionally were fortified by the study of Optimal Control, Stability and other notions, that follow.

The father of Automatic Control Theory is James Clerk Maxwell, who published in 1868 the text named "On Governors" with mathematical techniques for studying centrifugation, followed by Alexandr Mikhailovich Lyapunov in 1892, who published an article about the stability of moving objects, perfectly for that era till nowadays developed for simulating celestial objects, most prominently in our Solar System. The simulation that took place for the corresponding thesis is based on this fact.

Essentially at the same time as Maxwell conducted research on Control Theory, a former classmate ofhis, Edward John Routh made some significant work regarding moving forward several aspects of Maxwell'sresearch, which was utilized in World War I and II, in secret, though. Finally, more recently – in 1964 – Adolf Hurwitz made critical contribution in the field, by establishing a relation between the existence of non-negative eigenvalues of the linear functionals, which act on the automatic control system.

[1]And the BIBO (bounded input – bounded output) stability, which is the bounded analogue of Lyapunov stability, in input – output control systems

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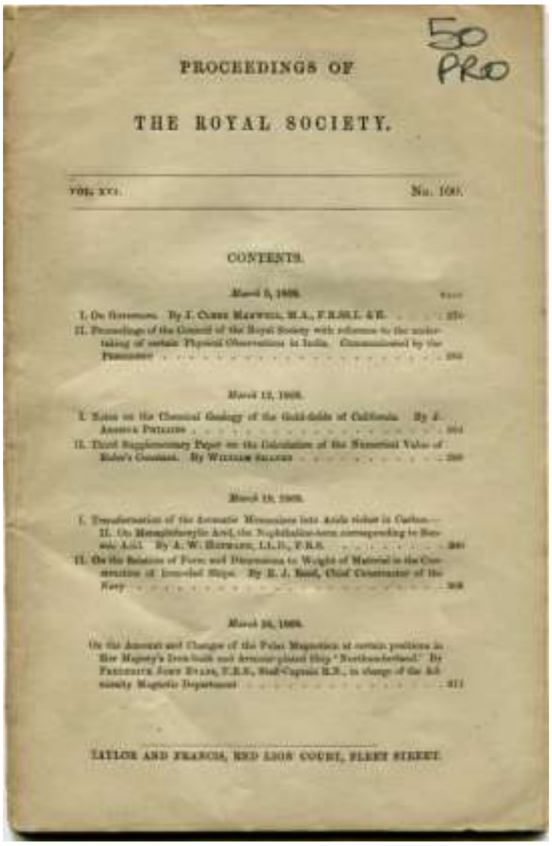
We would also like to pose that in the present article and in the corresponding Master's Thesis, one of the main aims was to fully understand the historic succession of events in order to develop a thorough comprehension of the theoretical structure and foundations behind modern Automatic Control Theory, or generally, Control Theory.



Picture 1: The first page of the journal publishing Maxwell's "On Governors"

B. Prerequisites

This subsection will address the reader to familiarize with some quite important facts about the theorybehind seemingly everything behind this article. Even more, some of the following are considered "heavymachinery".

1) Mathematics

The mathematical part in this article is of most importance, because it acts as the foundations upon whichwe build the rest and, obviously, the simulation. The reader can search in the following references the missinglinks.

Generally, we are concerned with the properties of the following system:

$$\begin{cases} \dot{y} = f(y,u) \\ y(0) = y_0 \end{cases}$$

where each$y(t) \in \mathbb{R}^n$ (state space) is the state of the system in the given time as a time – differentiable function, $y(0)$ is the initial value or state (constant vector), $u(t) \in \mathbb{R}^m$ is the control function and $f: \mathbb{R}^n \times \mathbb{R}^m = \mathbb{R}^{n+m} \longrightarrow \mathbb{R}^n$ is a continuous function.

It is quite common to write u as a feedback function, often useful in recursive algorithmic relations and methods, in the composition form

$$u = k \circ y$$

where $k: V \subset \mathbb{R}^n \longrightarrow \mathbb{R}^m$.

We also mention, that in the discrete case we solve and study the problem

$$y_{k+1} = y_k + f(y_k, u_k)$$

for$x(k) = x_k$, where$x \in \{y, u\}$.

a) Linearity

The original system is said to be linear, if $f$is linear, i.e. a linear operator, or a matrix operator.We write accordingly

$$\begin{cases} \dot{y} = Ay + Bu \\ y(0) = y_0 \end{cases}$$

where A and B are matrices of proper dimension and to be more specific $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$. In the usual case of linearity, the above k is written in matrix form as

$$u = Ky$$

which is followed by

$$\begin{cases} \dot{y} = Ay + BCy \\ y(0) = y_0 \end{cases} \equiv \begin{cases} \dot{y} = \mathcal{A}y \\ y(0) = y_0 \end{cases}$$

bysubstituting$A + BC = \mathcal{A}$, as the above being identical in terms of solution set. Most often it is written in the simplified form

$$\begin{cases} \dot{y} = Ay + a \\ y(0) = y_0 \end{cases}$$

by substituting $Bu = a$, which implies direct uniqueness for the following solution

$$y(t) = e^{tA}y_0 + \int_0^t e^{(t-s)A}a(t)ds$$

whereas simpler operations. The existence of the above solution in the appropriate closed interval $I \subset \mathbb{R}$ of the time variable is deduced by the Picard – Lindelöf Theorem. The closed interval in the set of real numbers is the key parameter of this theorem, because it encloses two notions of topological sets: "closed" and "bounded", which is identical in $\mathbb{R}$ to the – generally stronger – pair "complete" and "totally bounded", which in turn implies "compact". Compactness is a very powerful attribute of sets, and to be more specific, of topological or metric spaces, deducing a plethora of consequences, like above.

Linearity, also, deals with a number of interesting simplifications of the control system, borrowed by LinearAlgebra, as Gaussian elimination, triangulation and diagonalization, Jordan and Cholesky decomposition,QR and (P)LU decomposition, and many others, affecting the basis on which the matrix is expressed. These methods create a matrix,

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which is conjugate or similar to A, i.e. if $\mathcal{C}$ is the matrix formed by the above, then $A \sim \mathcal{C} \Leftrightarrow \mathcal{C} = UAU^{-1}$, for a matrix U ideal for triangulation, diagonalization and Jordan decomposition, where in the rest, we write for example $A = PLU$, for P, L and U being a permutation matrix, a lower triangular and an upper triangular, respectively.

b) Controlability

We pose a system like above as observable if for any two states z_1 $(t = 0)$, z_2 we have that there exists a $t^* \in [0, t_0]$ such that

$$y(t^*) = y_{z_1}^{z_2}(t^*)$$

where the symbol $y_{z_1}^{z_2}(t)$ means that

$$\begin{cases} y(0) = z_1 \\ y(t) = z_2 \end{cases}$$

and is called the flow of y from z_1 to z_2. In algebraic terms, we can use the adjoint matrices in order to come to the above characterization of the control system, using the Theorem of Ranks, namely the matrix

$$(A|B) = (B|AB|A^2B|\cdots|A^{n-1}B) = \begin{pmatrix} b_{11} & \cdots & b_{1m} & \sum_{i=1}^{n} a_{1i}b_{i1} & \cdots & \sum_{i=1}^{n} a_{1i}b_{im} & & \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & \cdots \\ b_{n1} & \cdots & b_{nm} & \sum_{i=1}^{n} a_{ni}b_{in} & \cdots & \sum_{i=1}^{n} a_{ni}b_{im} & & \end{pmatrix}$$

should have its rank equal to n.

c) Observability

It is common in automatic control systems or in dynamical systems not to full know the states $y(t)$ but to own partial knowledge or information of those, i.e. there is a function $h: \mathbb{R}^n \longrightarrow \mathbb{R}^l$ such that we observe

$$h \circ y$$

In the linear case, where h is linear, too, we have that we observe Hy, for H being the corresponding matrix.

d) Stability

Stability is a rather more general notion, but accompanied with linearity have a rich theory, leading to Lyapunov stability. An automatic control system or its corresponding/adjoint matrix is said to be stable or Lyapunov stable if

$$y(t) \xrightarrow{n \longrightarrow +\infty} 0$$

This analytic notion has an algebraic counterpart, namely, the Hurwitz criterion, using the eigen values of the corresponding matrix, i.e. the system is stable if

$$\omega(A) = \max_{\lambda \in \sigma(A)} \{Re(\lambda)\} \in (-1,0)$$

This criterion means that the real parts of the eigen values of the matrix of our control system should be greater than -1 and less than 0, in order for the system to be Lyapunov stable. The measure – theoretic or one stronger analytic version of stability needs

$$\int_0^{+\infty} |y(t)|^2 dt < +\infty$$

When our linear system depends on an input or an output and in order to keep in check with the problem – setting, we tend to ask for these functions to be bounded. So, the Lyapunov stability utilized to bounded input and output functions forms the BIBO stability.

Accordingly, in the non-linear case we utilize the equilibrium points$(\bar{y}, \bar{u})$ of f:

$$f(\bar{y}, \bar{u}) = 0$$

and then calculate the Jacobian matrix $\mathcal{J}_f$of the partial derivatives and seek local invertibility, local stability,observability, controlability, optimality or other properties – as in linear cases, now – in a small neighborhood of the equilibrium points. This method, together with the topological method of simply connected sets, the generalized Lyapunov functions and the energy functional method (variational method) are the major tools needed in studying ISS stability.

2) Computational Complexity

It is calculated that the computational time needed for testing the stability of a linear or linearized automatic control system, equipped with a matrix that can be weakly simplified – like in a Schur analysis – is

$$\mathcal{O}(n^3)$$

which is characterized as fast for being in the class of polynomial time algorithms, but in real life computingmay be needed a couple days.

3) Mechanics and Astronomy

The model we are approaching as a celestial mechanics complex example is actually a gravity simulation of an N – body situation. We pose some well – established facts about gravitational force. Let $\boldsymbol{F_G}$ be the gravitational vector field[2]. Then, this field is known to be pulling and central

$$\boldsymbol{F}_G(\boldsymbol{r}) = f(\boldsymbol{r}) \cdot \boldsymbol{r}$$

for$f: \mathbb{R}^3 \longrightarrow (-\infty, 0]$ and r be the position, while it is radially symmetric

$$\boldsymbol{F}_G(\boldsymbol{r}) = \widetilde{\boldsymbol{F}}_G(|\boldsymbol{r}|)$$

for$\widetilde{\boldsymbol{F}}_G: [0, +\infty) \longrightarrow \mathbb{R}^3$ to be the corresponding vector field that is expressed through the modulus$|\cdot|$ of the vectors. Finally, it is a conservative vector field, that is it is dependent in a certain motion only of the first and final state in space, which written in differential form

$$\boldsymbol{\nabla} \cdot \boldsymbol{F}_G = 0$$

[2] We use **bold** letters for vectors.

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Or equivalently

$$\frac{\partial F_{G,1}}{\partial x} + \frac{\partial F_{G,2}}{\partial y} + \frac{\partial F_{G,3}}{\partial z} = 0$$

for $\boldsymbol{F}_G = (F_{G,1}, F_{G,2}, F_{G,3})$, whilst in integral form we have

$$\oint \boldsymbol{F}_G \cdot d\boldsymbol{r} = 0$$

Or equivalently

$$\oint F_{G,1}dx + F_{G,2}dy + F_{G,3}dz = 0$$

This integral form suggests that in a close curve – a loop – this vector field needs no work to move a point of matter. The same holds in any conservative field, like the gravitational, as in the electric, but not in the magnetic.

The model of the Solar System which is simulated is characterized by a finite set of propositions, that simplifyin a justified manner, our effort:

- We will not take under account the tidal forces between the bodies.
- We will not re-calculate any miscalculation derived by the solar wind.
- We will assume that the radii of the bodies included in the simulation are significantly smaller than
- the radius of their orbit, or even compared to the Solar System's radius.
- We will use as initial value circular and not elliptic orbits.
- We assume that a normalized contemporary Titius-Bode Law applies.
- We use the Sun, the Moon and the first 7 planets.

II. MODEL

A. *Python3 Coding*

The Python3 code written and used for the simulation can retrieved here:

```
        #! /usr/bin/env python3

importmath
fromturtleimport*

G=6.67428e-11

AU= (149.6e6*1000)
SCALE=250/AU

classBody(Turtle):
"""
    Subclass   of   Turtle   representing   a
gravitationally-acting body.

    Extra attributes:
    mass: mass in kg
vx, vy: x, y velocities in m/s
px, py: x, y positions in m
    """

name='Body'
mass=None
vx=vy=0.0
px=py=0.0

defattraction(self, other):
"""
        (Body): (fx, fy)

        Returns  the  force  exerted  upon  this
body by the other body.
        """

ifselfisother:
raiseValueError("Attraction  of  object  %r  to
itself requested"
%self.name)


sx, sy=self.px, self.py
ox, oy=other.px, other.py
dx= (ox-sx)
dy= (oy-sy)
d=math.sqrt(dx**2+dy**2)


ifd==0:
raiseValueError("Collision  between  objects  %r
and %r"
% (self.name, other.name))

f=G*self.mass*other.mass/ (d**2)


theta=math.atan2(dy, dx)
fx=math.cos(theta) *f
fy=math.sin(theta) *f
returnfx, fy

defupdate_info(step, bodies):
"""
    (int, [Body])

    Displays  information  about  the  status  of
the simulation.
    """
print('Step #{}'.format(step))
forbodyinbodies:
s='{:<8}          Pos.={:>6.2f}      {:>6.2f}
Vel.={:>10.3f} {:>10.3f}'.format(
body.name,   body.px/AU,   body.py/AU,   body.vx,
body.vy)
print(s)
print()

defloop(bodies):
"""
    ([Body])
```

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```
    Never returns; loops through the simulation, updating the
    positions of all the provided bodies.
    """
timestep=24*3600

forbodyinbodies:
body.penup()
body.hideturtle()

step=1
whileTrue:
update_info(step, bodies)
step+=1

force= {}
forbodyinbodies:

total_fx=total_fy=0.0
forotherinbodies:

ifbodyisother:
continue
fx, fy=body.attraction(other)
total_fx+=fx
total_fy+=fy


force[body] = (total_fx, total_fy)


forbodyinbodies:
fx, fy=force[body]
body.vx+=fx/body.mass*timestep
body.vy+=fy/body.mass*timestep


body.px+=body.vx*timestep
body.py+=body.vy*timestep
body.goto(body.px*SCALE, body.py*SCALE)
body.dot(3)


defmain():

sun=Body()
sun.name='Sun'
sun.mass=1.98892*10**30
sun.pencolor('yellow')

earth=Body()
earth.name='Earth'
earth.mass=5.9742*10**24
earth.px=1*AU
earth.vy=29.783*1000# 29.783 km/sec
earth.pencolor('blue')

  moon=Body()
moon.name='Moon'
moon.mass=7.342*10**22
moon.px=1.002*AU
moon.vy=29.783*1000
moon.pencolor('grey')

  mercury=Body()
mercury.name='Mercury'
mercury.mass=3.302*10**23
mercury.px=0.387*AU
mercury.vy=47.87*1000
mercury.pencolor('red')


  venus=Body()
venus.name='Venus'
venus.mass=4.8685*10**24
venus.px=0.723*AU
venus.vy=35.02*1000
venus.pencolor('orange')

  mars=Body()
mars.name='Mars'
mars.mass=6.4191*10**23
mars.px=1.524*AU
mars.vy=24.13*1000
mars.pencolor('red')

  jupiter=Body()
jupiter.name='Jupiter'
jupiter.mass=1.8987*10**27
jupiter.px=5.203*AU
jupiter.vy=13.07*1000
jupiter.pencolor('pink')

  saturn=Body()
saturn.name='Saturn'
saturn.mass=9.5371*10**26
saturn.px=5.203*AU
saturn.vy=9.67*1000
saturn.pencolor('white')


loop([sun, earth, moon, mercury, venus, mars, jupiter, saturn])

if __name__ =='__main__':
main()
```

B. Snapshots

In this subsection, we are going to present 4 main snapshots of the simulation.

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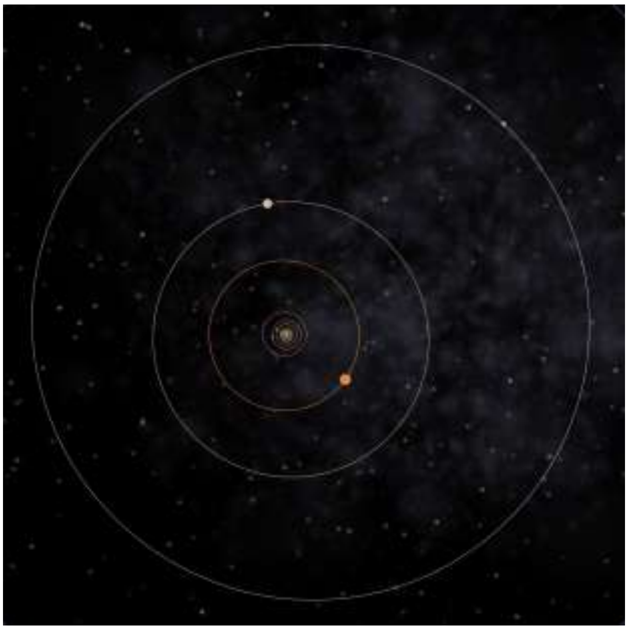
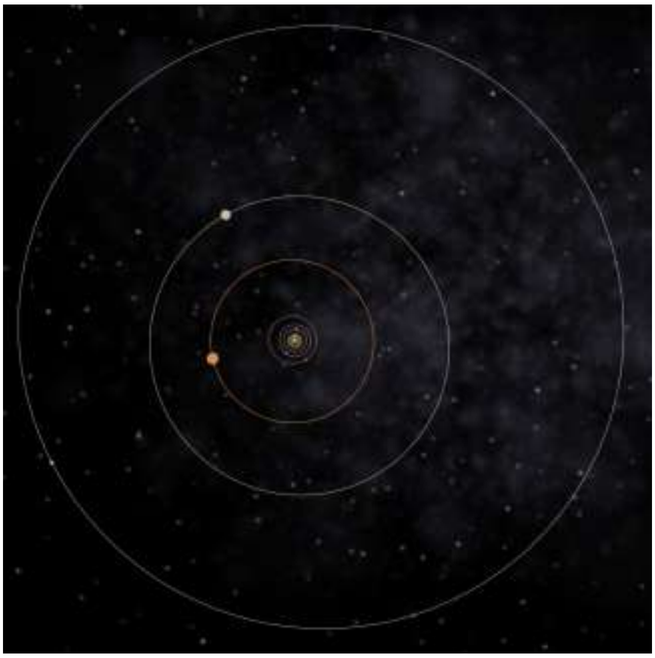


Picture 2: ~90My from formation or ~4.477Gy in the past

The first million years (My – Mega years) after the Solar System formation, from an initial status of interstellar nebula. We also note that the abbreviation Gy (Giga years) stands for billion years.

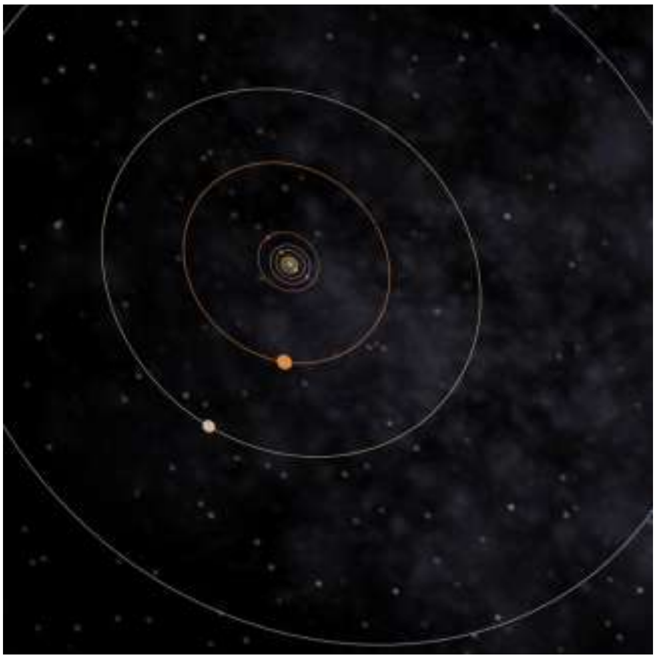


Picture3: 0.873Gy in the future

Snapshot of an approximately modern picture.

Picture 4: 2.467Gy in the future

In about several Gy from today, the orbits are going to reformulate and be reshaped under the effect of non-existing stability.



Picture 5: ~5.1Gy in the future

In about 4 to 5.5 Gy from now, the orbits are totally changed.

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III. RESULTS

A. Modelling results

The modelling results suggest that the Solar System will be in a Lyapunov stable form for about 1Gy from today, while then, it will face a critical change in stability due to the ongoing effect of celestial mechanics laws, which tend to decompose the much more "perfect" initial view. Instability begins after that critical point in space time and the inner planets are going to rapidly spin around the Sun, till they crash on its surface, which by then will have been expanded. The outer planets, will be discarded to parabolic, hyper bolicor cubic orbits out of the Solar System.

Concluding, the main result is

$$2 \leq d \longrightarrow 3$$

for d being the algebraic dimension of the curve that simulates the body's orbit, as

$$\rho \uparrow +\infty$$

where ρ is the coefficient corresponding to Lyapunov stability, which begins in the stability domain and approaches instable states and, finally, in infinite time deterministic chaotic behavior.

B. Survey results

The corresponding thesis includes a survey that took place among certain people, mixed with either or not academic credentials, by wide set of disciplines. In order to be more precise, the survey was conducted with the help of 2 Automation Engineers, 2 Mathematicians, 3 Mechanical Engineers, 1 Electrical Engineer, 1 Electronics Engineer and 1 Auto Technician, to the total of 10 interviewers.

Σπουδές/Επάγγελμα

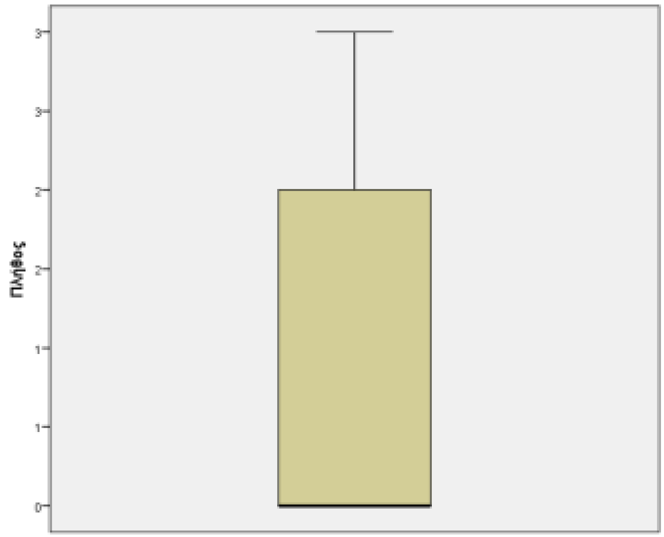
		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Ηλεκτρολόγος Μηχανικός	1	10,0	10,0	10,0
	Ηλεκτρονικός	1	10,0	10,0	20,0
	Μαθηματικός	2	20,0	20,0	40,0
	Μηχανικός Αυτοματισμού	2	20,0	20,0	60,0
	Μηχανολόγος Μηχανικός	3	30,0	30,0	90,0
	Τεχνικός Αυτοκινήτων	1	10,0	10,0	100,0
	Total	10	100,0	100,0	

Picture 6: Interviewers' background (in Greek)

In the questionnaire, there has been a total of 5 questions, asking for a Yes – No answer, but also, there were some other possible answers, like the number of stabilities known to the interviewers, which was a of certain interest. The mean was 0.8 known stabilities in a confidence interval of

$$[0, 1.935]$$

and a box plot characterized by the 50% of the interviewers to hold none known stability as shown (bold line)



Picture 7: Boxplot for total of known stabilities (in Greek)

What should be absolutely mentioned is that the interviewers who applied positively in both question 2 and 3 (knowledge versus simple usage/utilization/application of stabilities in control systems) were the most likely to do so. The reference is the crosstabulation here.

Chi-Square Tests

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	$10{,}000^a$	1	,002		
Continuity Correction[b]	6,267	1	,012		
Likelihood Ratio	13,460	1	,000		
Fisher's Exact Test				,005	,005
N of Valid Cases	10				

a. 4 cells (100,0%) have expected count less than 5. The minimum expected count is 1,60.
b. Computed only for a 2x2 table

Picture 8: Crosstabulation showing positive relation between questions 2 and 3

The likelihood ratio is over 13, which is almost 40% more than the next greater ratio.

The main results were that these professionals have heard theoretical terms about Control Theory, though not all in a way that can be useful in comparing Lyapunovto BIBO stability, Lyapunov to ISS stability and, finally, BIBO to ISS stability. On their account, though, we should pose that Lyapunov stability was somewhat known, even in a simplified computer – assisted version.

The above facts were extrapolated by a set of statistical measures and techniques[3] utilized and an oral interview.

[3]Measures like mean, median, leading values, quartiles and more, while techniques like graphical methods (pies, boxplots etc.) and crosstabulations.

C. *Further research*

The author suggests further research on the following:

- Studying of the stability of non–linear inputs from Signal Theory and Processing.
- Studying ISS stability compared to weak notions of observability, controlability and realization.
- Studying topological aspects of nonlinear systems emphasizing in simply and non-simply connected sets.
- Studying stochastic chaos in probability driven control systems using Malliavin Calculus.

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