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THERMO-MECHANICAL INTERACTIONS DUE TO ULTRA THERMAL LASER SOURCE IN MICROPOLAR THERMOELASTIC DIFFUSIVE HALF SPACE

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***Abstract*—The present investigation deals with the deformation in micro polar generalized thermo elastic medium with mass diffusion subjected to thermo mechanical loading due to thermal laser pulse. Laplace and Fourier transform technique is used to solve the problem. Concentrated normal force and thermal source are taken to illustrate the utility of approach. The closed form expressions of normal stress, tangential stress, tangential couple stress, mass concentration and temperature distribution are obtained in the transformed domain. Numerical inversion technique of Laplace transform and Fourier transform has been applied to obtain the resulting quantities in the physical domain after developing a computer program. The normal stress, tangential stress, tangential coupled stress, temperature distribution and mass concentration are depicted graphically to show the effect of relaxation times. Some particular cases of interest are deduced from the present investigation.**

Keywords—Laser Pulse, Micro polar, Mass diffusion, uniformly and linearly distributed source.

I. INTRODUCTION

Micro polar theory of elasticity was introduced by Eringen (1966). This theory incorporates the local deformation and rotation of the material points of the composite. This theory provides a model that can support body couples and surface couples and exhibits a high frequency optical wave spectrum. Eringen (1971, 1999), Maugin and Mild (1986), Nowacki (1970) developed the linear theory of micro polar thermo elasticity by excluding the micro polar theory of elasticity to include the thermal effects. Touchert et al. (1968), derived the basic equations of linear theory of micro polar coupled thermo elasticity.

Diffusion is defined as the spontaneous movement of the particles from a high concentration region to the low-concentration region, and it occurs in response to a concentration gradient expresses as the change in the concentration due to change in position. Thermal diffusion utilizes the transfer of heat across a thin liquid or gas to accomplish isotope separation. Today, thermal diffusion remains a practical process to separate isotopes of noble gases e.g., Xenon and other light isotopes e.g., Carbon for research purposes. In most of the applications, the concentration is calculated using Fick's law. This is a simple law which does not take into consideration the mutual interaction between the introduced substance and the medium into which it is introduced or the effect of temperature of this interaction. However, there is a certain degree of coupling with temperature and temperature gradients as temperature speeds up the diffusion process. Nowacki (1974, 1976) developed the theory of thermo elastic diffusion by using coupled thermo elastic model. Dudziak and Kowalski and Olesiak and Pyryev (1995), respectively, discussed the theory of thermo diffusion and coupled quasi stationary problems of thermal diffusion for an elastic layer.

Laser technology has a vital application in nondestructive materials testing and evaluation. When a solid is heated with a laser pulse, it absorbs some energy which results in an increase in localized temperature. This cause thermal expansion and generation of the ultrasonic waves in the material. There are generally two mechanisms for such wave generation, depending on the energy density deposited by the laser pulse. At high energy density, a thin surface layer of the solid material melts, followed by an ablation process whereby particles fly off the surface, thus giving rise to forces that generates ultrasonic waves. At low energy density, the surface material does not melt, but it expands at a high rate and wave and wave motion is generated due to thermo elastic processes.

Very rapid thermal processes (e.g., the thermal shock due to exposure to an ultra-short laser pulse) are interesting from the stand point of thermo elasticity, since they require a coupled analysis of the temperature and deformation fields. A thermal shock induces very rapid movement in the structural elements, giving the rise to very significant inertial forces, and thereby, an increase in vibration. Rapidly oscillating contraction and expansion generates temperature changes in materials susceptible to diffusion of heat by conduction (1999). This mechanism has attracted considerable attention due to the extensive use of pulsed laser technologies in material processing and non-destructive testing and characterization (2001, 2002). The so-called ultra short lasers are those with pulse durations ranging from nanoseconds to femto seconds. In the case of ultra short pulsed laser heating, the high intensity energy flux and ultra short duration lead to a very large thermal gradients or ultra-high heating may exist at the boundaries. In such cases, as pointed out by many investigators, the classical Fourier model, which leads to an infinite propagation speed of the thermal energy, is no longer valid (1989). Researchers have proposed several models to describe the mechanism of heat conduction during short-pulse laser heating, such as the parabolic one-step model (1994), the hyperbolic one-step model (1990), and the parabolic two-step and hyperbolic two-step models (1993, 1997).

Dubois (1994) experimentally demonstrated that penetration depth play a very important role in the laser-ultrasound generation process. Ezzat et al. (2012) discussed the thermo-elastic behavior in metal films by fractional ultrafast laser. Al-Huniti and Al-Nimr (2004) investigated the thermo elastic behavior of a composite slab under a rapid dual-phase lag heating. The comparison of one-dimensional and two-dimensional axisymmetric approaches to the thermo mechanical response caused by ultra short laser heating was studied by Chen et al. (2002).Kim et al.(1997) studied thermo elastic stresses in a bonded layer due to pulsed laser radiation. Thermo elastic material response due to laser pulse heating in context of four theorems of thermo elasticity was discussed by Youssef and Al-Bary (2014),Theoretical study of the effect of enamel parameters on laser induced surface acoustic waves in human incisor was studied by Yuan et al (2014).A two-dimensional generalized thermo elastic diffusion problem for a thick plate under the effect of laser pulse thermal heating was studied by Elhagary (2014).Othman et al. (2014)studied the influence of thermal loading due to laser pulse on generalized micro polar thermo elastic solid with comparison of different theories. The exact analysis of laser generated thermo elastic waves in an anisotropic infinite plate mathematically done by Al- Qahtani and Datta (2008).Deswal, Sheoran and Kalkal (2013)investigated a two-dimensional problem in magnetothermoelasticity with laser pulse under different boundary conditions.

In this research, taking into account the mass concentration effect and radiation of ultra short laser, we have established a model for micro polar thermo elastic medium with mass diffusion by using Laplace and Fourier transforms. The stress components and temperature distribution have been computed numerically. The resulting quantities are shown graphically to show the effect of mass concentration and temperature.

II.PROBLEM FORMULATION

Following Eringen (1999), Sherief (2004) and Al-Qahtani and Datta (2008) the basic equations for homogeneous, isotropic micro polar generalized thermo elastic solid with mass diffusion in the absence of body forces and body couples are given by:

$$(\lambda+\mu)\nabla(\nabla.\boldsymbol{u})+(\mu+K)\nabla^2\boldsymbol{u}+K\nabla\times\boldsymbol{\phi}-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla C=\rho\ddot{\boldsymbol{u}},(1)$$

$$(\gamma\nabla^2-2K)\boldsymbol{\phi}+(\alpha+\beta)\nabla(\nabla.\boldsymbol{\phi})+K\nabla\times\boldsymbol{u}=\rho j\ddot{\boldsymbol{\phi}},(2)$$

$$K^*\nabla^2T=\rho c^*\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+\left(1+\varepsilon\tau_0\frac{\partial}{\partial t}\right)(\beta_1T_0\nabla.\dot{\boldsymbol{u}}-Q)+aT_0\left(\frac{\partial}{\partial t}+\gamma_1\frac{\partial^2}{\partial t^2}\right)C,(3)$$

$$D\beta_2\nabla^2(\nabla.\boldsymbol{u})+Da\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla^2T+\left(\frac{\partial}{\partial t}+\varepsilon\tau^0\frac{\partial^2}{\partial t^2}\right)C-Db\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla^2C=0,(4)$$

$$t_{ij}=\lambda u_{k,k}\delta_{ij}+\mu(u_{i,j}+u_{j,i})+K(u_{j,i}-\epsilon_{ijk}\phi_k)-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\delta_{ij}T-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\delta_{ij}C, \qquad (5)$$

$$m_{ij}=\alpha\phi_{k,k}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}, \qquad (6)$$

The plate surface is illuminated by laser pulse given by the heat input

$$Q=I_0f(t)g(x_1)h(x_3) \qquad (7)$$

where I_0 is the energy absorbed. The temporal profile $f(t)$ is represented as,

$$f(t)=\frac{t}{t_0^2}e^{-\left(\frac{t}{t_0}\right)} \qquad (8)$$

Here t_0 is the pulse rise time. The pulse is also assumed to have a Gaussian spatial profile in x_1

$$g(x)=\frac{1}{2\pi r^2}e^{-\left(\frac{x_1^2}{r^2}\right)} \qquad (9)$$

where r is the beam radius, and as a function of the depth x_3 the heat deposition dueto the laser pulse is assumed to decay exponentially within the solid,

$$h(x_3)=\gamma^*e^{-\gamma^*x_3} \qquad (10)$$

Equation (7) with the aid of (8,9 and 10) takes the form

$$Q = \frac{I_0\gamma^*}{2\pi r^2 t_0^2} t e^{-\left(\frac{t}{t_0}\right)} e^{-\left(\frac{x_1^2}{r^2}\right)} e^{-\gamma^* x_3} \quad , \tag{11}$$

where λ, μ, α, β, γ, K, are material constants, ρ is mass density, $\boldsymbol{u} = (u_1, u_2, u_3)$is the displacement vector and $\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3)$is the microrotation vector, T is temperature and T_0 is the reference temperature of the body chosen,K^* is the coefficient of the thermal conductivity,c^* is the specific heat at constant strain, D is the thermoelastic diffusion constant, a is the coefficient describing the measure of thermo diffusion and b is the coefficient describing the measure of mass diffusion effects,j is the microinertia, $\beta_1 = (3\lambda + 2\mu + K)\alpha_{t1}$,$\beta_2 = (3\lambda + 2\mu + K)\alpha_{c1}$, $\alpha_{t1}, \alpha_{c1}$are coefficients of linear thermal expansion and coefficients of linear diffusion expansion,t_0 is the pulse rise time,I_0 is the energy absorbed,t_{ij} are components of stress vector, m_{ij} are components of couple stress vector, δ_{ij} is Kroneker delta function, $\tau_0, \tau_1$are thermal relaxation times with $\tau_0 \geq \tau_1 \geq 0$.

We consider a micro polar generalized thermo elastic mass diffusion medium with rectangular Cartesian coordinate system $OX_1X_2X_3$ having origin on x_3-axis with x_3-axis pointing vertically inward the medium.

We consider plane strain problem with all the field variables depending on$x_1, x_3$and t. For two dimensional problems, we take

$$\boldsymbol{u} = (u_1, 0, u_3), \boldsymbol{\phi} = (0, \phi_2, 0), \tag{12}$$

For further consideration, it is convenient to introduce in equations (1.1)-(1.4) the dimensionless quantities defined as:

$$u_i' = \frac{\rho\omega^* c_1}{\beta_1 T_0} u_i \ , \quad x_i' = \frac{\omega^*}{c_1} x_i \ , \quad t' = \omega^* t \ , \quad T' = \frac{T}{T_0} \ , \quad \tau_1' = \omega^*\tau_1 \ , \quad \tau_0' = \omega^*\tau_0 \ , \gamma_1' = \omega^*\gamma_1, \ t_{ij}' = \frac{1}{\beta_1 T_0} t_{ij} \ , \omega^* = \frac{\rho c^* c_1^2}{K^*}, \phi_i' = \frac{\rho c_1^2}{\beta_1 T_0}\phi_i \ , \quad \tau^{1\prime} = \omega^*\tau^1 \ , \quad c_1^2 = \frac{\lambda + 2\mu + k}{\rho}, \quad c_2^2 = \frac{\mu + k}{\rho} \ , \quad c_3^2 = \frac{\gamma}{\rho j} \ , \quad c_4^2 = \frac{2\alpha_0}{\rho j_0} \ , \quad \varepsilon = \frac{\gamma^2 T_0}{\rho^2 c^* c_1} \ , m_{ij}^* = \frac{\omega^*}{c\beta_1 T_0} m_{ij} \ , \quad C' = \frac{\beta_2}{\rho c_1^2} C \ , \ Q = \frac{K^*{\omega^*}^2}{C^*} Q' \tag{13}$$

Making use of (13) in (1)-(4) and with the aid of (12), we obtain:

$$a_1 \frac{\partial e}{\partial x_1} + a_2\nabla^2 u_1 - a_3\frac{\partial \phi_2}{\partial x_3} - \left(1 + \tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1} - a_4\left(1 + \tau^1\frac{\partial}{\partial t}\right) C = \rho\frac{\partial^2 u_1}{\partial t^2}, \tag{14}$$

$$a_1 \frac{\partial e}{\partial x_3} + a_2\nabla^2 u_3 + a_3\frac{\partial \phi_2}{\partial x_1} - \left(1 + \tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_3} - a_4\left(1 + \tau^1\frac{\partial}{\partial t}\right) C = \rho\frac{\partial^2 u_3}{\partial t^2}, \tag{15}$$

$$\nabla^2\phi_2 - 2a_6\phi_2 + a_6\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) = a_7\ddot{\phi}_2 \tag{16}$$

$$-\nabla^2 T + \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right) T + a_5\left(\frac{\partial}{\partial t} + \varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right) e + a_8\left(\frac{\partial}{\partial t} + \gamma_1\frac{\partial^2}{\partial t^2}\right) C = Q_0 f^*(x_1, t) e^{-\gamma^* x_3} \tag{17}$$

$$\nabla^2 e + a_9\left(1 + \tau_1\frac{\partial}{\partial t}\right)\nabla^2 T + a_{10}\left(1 + \epsilon\tau^0\frac{\partial}{\partial t}\right)\dot{C} - a_{11}\left(1 + \tau^1\frac{\partial}{\partial t}\right)\nabla^2 C = 0 \ , \tag{18}$$

where, $a_1 = \frac{\lambda+\mu}{\rho c_1^2}$, $a_2 = \frac{\mu+K}{\rho c_1^2}$, $a_3 = \frac{K}{\rho c_1^2}$, $a_4 = \frac{\rho c_1^2}{\beta_1 T_0}$, $a_6 = \frac{K c_1^2}{\gamma {\omega^*}^2}$, $a_7 = \frac{\rho j c_1^2}{\gamma}$, $a_5 = \frac{\beta_1^2 T_0}{\rho K^* \omega^*}$, $a_8 = \frac{a\rho c_1^4}{\omega^* \beta_2 K^*}$, $a_9 = \frac{a\rho c_1^2}{\beta_1\beta_2}$, $a_{10} = \frac{\rho c_1^4}{D\omega^*\beta_2^2}$, $a_{11} = \frac{b\rho c_1^2}{\beta_2^2}$, $Q_0 = \frac{a_{13} I_0\gamma^*}{2\pi r^2 t_0^2}$

$$f(x_1, t) = \left[t + \epsilon\tau_0\left(1 - \frac{t}{t_0}\right)\right] e^{-\left(\frac{x_1^2}{r^2} + \frac{t}{t_0}\right)}$$

The displacement components u_1 and u_3 are related to the non- dimensional potential functions ϕ and ψ as:

$$u_1 = \frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3} \ , \quad u_3 = \frac{\partial\phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1} \tag{19}$$

Substituting the values of $u_1$andu_3 from (19) in (14)-(18) and with the aid of (12), we obtain:

$$\nabla^2\phi - \ddot{\phi} - \left(1 + \tau_1\frac{\partial}{\partial t}\right) T - a_4\left(1 + \tau^1\frac{\partial}{\partial t}\right) C = 0, \tag{20}$$

$$\nabla^4\phi + a_9\left(1 + \tau_1\frac{\partial}{\partial t}\right)\nabla^2 T + a_{10}\left(1 + \epsilon\tau^0\frac{\partial}{\partial t}\right)\dot{C} - a_{11}\left(1 + \tau^1\frac{\partial}{\partial t}\right)\nabla^2 C = 0, \tag{21}$$

$$\left(1 + \tau_0\frac{\partial}{\partial t}\right)\dot{T} + a_5\left(\frac{\partial}{\partial t} + \varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\nabla^2\phi + a_8\left(\frac{\partial}{\partial t} + \gamma_1\frac{\partial^2}{\partial t^2}\right) C - \nabla^2 T = Q_0 f^*(x_1, t) e^{-\gamma^* x_3}, \tag{22}$$

$$a_2\nabla^2\psi - \ddot{\psi} + a_3\phi_2 = 0, \tag{23}$$

$$\nabla^2\phi_2 - 2a_6\phi_2 - a_6\nabla^2\psi = a_7\ddot{\phi}_2, \tag{24}$$

III. SOLUTION OF THE PROBLEM

We define Laplace transform and Fourier transform respectively as:

$$\bar{f}(s, x_1, x_3) = \int_0^\infty f(t, x_1, x_3) e^{-st} dt \ , \tag{25}$$

$$\hat{f}(x_3, \xi, s) = \int_{-\infty}^{\infty} \bar{f}(s, x_1, x_3) e^{\iota\xi x_1} dx_1 \ , \tag{26}$$

Applying Laplace transform defined by (25) on (20)-(24) and then applying Fourier transforms defined by (26) on the resulting quantities and eliminating$\hat{C}$&$\hat{T}$, $\hat{\phi}$&$\hat{T}$,$\hat{\phi}$&$\hat{C}$ and $\hat{\phi}_2$ respectively from the resulting equations, we obtain:

$$[D^6 + AD^4 + BD^2 + C]\hat{\phi} = f_1 e^{-\gamma^* x_3} \quad (27)$$

$$[D^6 + AD^4 + BD^2 + C]\hat{T} = f_2 e^{-\gamma^* x_3} \quad (28)$$

$$[D^6 + AD^4 + BD^2 + C]\hat{C} = f_3 e^{-\gamma^* x_3} \quad (29)$$

$$[D^4 + ED^2 + F]\hat{\psi} = 0\,, \quad (30)$$

where

$$A = -\frac{(a_{34}a_{37} + a_{35} + a_{40} + a_{36}a_{42})}{a_{39}}, B = -\frac{(a_{34}(a_{38} - a_{37}\xi^2) + a_{36}a_{43} - a_{40} - a_{41})}{a_{39}}, C = -(-a_{34}a_{38}\xi^2 + a_{41}a_{35} + a_{36}a_{44})/a_{39}\,, E = \frac{a_{31}}{a_2}, F = \frac{a_{32}}{a_2}, \tau_{11} = (1 + \tau_1 s), \xi_1 = \xi^2 + s^2, f_1 = Q_1(a_{37}\gamma^{*2} + a_{38})/a_{39}\,,$$

$$f_2 = Q_1(a_{39}\gamma^{*4} - a_{40}\gamma^{*2} + a_{41})/a_{39}\,, f_3 = Q_1(a_{42}\gamma^{*4} + a_{43}\gamma^{*2} + a_{44})/a_{39}\,, f_4 = [\gamma^{*6} + A\gamma^{*4} + B\gamma^{*2} + C]$$

and

$$a_{12} = a_9\tau_{11}\,, a_{13} = a_{10}(s + \varepsilon s^2\tau^0)\,, a_{14} = a_{11}(1 + \tau^1 s)\,, a_{15} = a_4(s + \varepsilon s^2\tau_0)\,, a_{16} = s + \xi^2 + s^2\tau_0, a_{17} = a_8(s + \gamma_1 s^2)\,, a_{18} = \xi^2 + 2a_6 + s^2 a_7\,,$$

The solutions of the equations (27)-(30) satisfying the radiation conditions that $(\hat{\phi}, \widehat{\phi^*}, \hat{T}, \widehat{\phi_2}, \hat{\psi}) \to 0$ as $x_3 \to \infty$ are given by:

$$\hat{\phi} = B_1 e^{-m_1 x_3} + B_2 e^{-m_2 x_3} + B_3 e^{-m_3 x_3} + L_1 e^{-\gamma^* x_3} \quad (31)$$

$$\hat{T} = d_1 B_1 e^{-m_1 x_3} + d_2 B_2 e^{-m_2 x_3} + d_3 B_3 e^{-m_3 x_3} + L_2 e^{-\gamma^* x_3}\,. \quad (32)$$

$$\hat{C} = e_1 B_1 e^{-m_1 x_3} + e_2 B_2 e^{-m_2 x_3} + e_3 B_3 e^{-m_3 x_3} + L_3 e^{-\gamma^* x_3}\,. \quad (33)$$

$$\hat{\psi} = B_4 e^{-m_4 x_3} + B_5 e^{-m_5 x_3} \quad (34)$$

$$\widehat{\phi_2} = h_4 B_4 e^{-m_4 x_3} + h_5 B_5 e^{-m_5 x_3} \quad (35)$$

where

$$d_i = \frac{a_{39}m_i^4 - a_{40}m_i^2 + a_{41}}{a_{37}m_i^2 + a_{38}}, e_i = \frac{a_{42}m_i^4 + a_{48}m_i^2 + a_{44}}{a_{37}m_i^2 + a_{38}}\,, \quad i = 1,2,3$$

$$\& h_i = \frac{a_2(m_i^2 - \xi_1)}{a_3}\,, i = 5,6$$

$$L_i = \frac{f_i}{[m_i^6 + Am_i^4 + Bm_i^2 + C]}, i = 1,2,3$$

and $m_i^2 (i = 1,2,3)$ are the roots of the characteristic equation of equation (27) and $m_i^2 (i = 4,5)$ are the roots of the characteristic equation of equation (30).

IV.BOUNDARY CONDITIONS

We consider concentrated normal force and concentrated thermal source at the boundary surface $x_3 = 0$, mathematically, these can be written as:

$$t_{33} = -F_1\psi_1(x_1)\delta(t),,$$
$$t_{31} = 0,$$
$$m_{32} = 0,$$
$$T = F_2\psi_1(x_1)\delta(t)\,,$$
$$C = F_3\psi_1(x_1)\delta(t) \quad (36)$$

where F_1 is the magnitude of the applied force and F_2 is the constant temperature applied on the boundary.

Also

$$t_{33} = \lambda e + (2\mu + K)u_{3,3} - \beta_1\left(1 + \tau_1\frac{\partial}{\partial t}\right)T - \beta_2\left(1 + \tau^1\frac{\partial}{\partial t}\right)C$$
$$t_{31} = (2\mu + K)u_{3,1} - K\phi_2$$
$$m_{32} = \beta\phi_{2,3} \quad (37)$$

Substituting the values of $\hat{\phi}, \widehat{\phi^*}, \hat{T}, \hat{\psi}, \widehat{\phi_2}$ from the equations (31)-(35) in the boundary condition (36) and using (5)-(11), (12)-(13), (25)-(26) and solving the resulting equations, we obtain:

$$\widehat{t_{33}} = \sum_{i=1}^{5} G_{1i}e^{-m_i x_3} - M_1 e^{-\gamma^* x_3}, i = 1,2,\ldots,5 \quad (38)$$

$$\widehat{t_{31}} = \sum_{i=1}^{5} G_{2i}e^{-m_i x_3} - M_2 e^{-\gamma^* x_3}, i = 1,2,\ldots,5 \quad (39)$$

$$\widehat{m_{32}} = \sum_{i=1}^{5} G_{3i}e^{-m_i x_3} - M_3 e^{-\gamma^* x_3}, i = 1,2,\ldots,5 \quad (40)$$

$$\hat{T} = \sum_{i=1}^{5} G_{4i}e^{-m_i x_3} - M_4 e^{-\gamma^* x_3}\,, i = 1,2,\ldots,5 \quad (41)$$

$$\hat{C} = \sum_{i=1}^{5} G_{5i}e^{-m_i x_3} - M_5 e^{-\gamma^* x_3}, i = 1,2,\ldots,5 \quad (42)$$

where

$$G_{mi} = g_{mi}C_i\,, C_i = \frac{\Delta_i}{\Delta_0}, \; i = 1,2,\ldots,5$$

$$g_{1i} = (m_i^2 - b_2\xi^2) - (1 + \tau_1 s)\alpha_{1i} - b_{11}\alpha_{2i}(1 + \tau^1 s)$$

$$g_{2i} = (b_5 + b_6)\iota\xi \quad, g_{3i} = 0 \quad, \; g_{4i} = \alpha_i, \; g_{5i} = m_i\beta_i, \quad i = 1,2,3$$

$g_{1l} = -\iota\xi b_3 m_l$, $g_{2l} = (b_6 m_l^2 + b_5\xi^2) - b_7\alpha_{3l}$, $g_{3l} = -b_8 m_l \alpha_{3l}$, $g_{4l} = 0$, $g_{5l} = 0, l = 4,5$

$$\Delta_0 = \begin{vmatrix} g_{11} & g_{12} & g_{13} & g_{14} & g_{15} \\ g_{21} & g_{22} & g_{23} & g_{24} & g_{25} \\ g_{31} & g_{32} & g_{33} & g_{34} & g_{35} \\ g_{41} & g_{42} & g_{43} & g_{44} & g_{45} \\ g_{51} & g_{51} & g_{51} & g_{51} & g_{51} \end{vmatrix},$$

$\Delta_1, \Delta_2, \Delta_3, \Delta_4 \& \Delta_5$ are obtained by replacing 1st ,2nd ,3rd ,4th and 5th columns by

$$\left[\left(M_1 + F_1\widehat{\psi_1}(s)\right),\ M_2\ ,\ M_3\ ,\ \left(M_4 - F_1\widehat{\psi_1}(s)\right)\ ,\ M_5\right]' \quad \text{in}\Delta_0$$

and

$$M_1 = -\left(\frac{\left(\gamma^{*2} - b_2\xi^2\right)f_1 - (1+\tau_1 s)f_2 - b_{11}(1+\tau^1 s)f_3}{f_4}\right),$$

$$M_2 = -\frac{(b_5+b_6)\iota\xi\gamma^* f_1}{f_4}, M_3 = 0,\ M_4 = -\frac{\gamma^* f_2}{f_4}\ ,\ M_5 = -\frac{\gamma^* f_3}{f_4}$$

Case 1: for the thermal source:$F_1 = 0$

Case 2: for the normal source:$F_2 = 0$

V.APPLICATIONS

(a) Uniformly distributed source:

The solution due to uniformly distributed force applied on the half-space is obtained by setting

$$\psi_1(x_1) = \begin{cases} 1, & |x_1| \le d \\ 0, & |x_1| > d \end{cases} \qquad (43)$$

Applying Laplace and Fourier transforms on (4.7), gives

$$\widehat{\psi_1}(\xi) = \frac{2\sin(\xi d)}{\xi}, \xi \ne 0 \qquad (44)$$

(b) Linearly distributed source:

The solution due to linearly distributed force over a strip of non-dimensional width 2d, applied on the half-space is obtained by setting

$$\psi_1(x_1) = \begin{cases} 1 - \frac{|x_1|}{d}, & |x_1| \le d \\ 0, & |x_1| > d \end{cases} \qquad (45)$$

Applying Laplace and Fourier transforms on (4.7), gives

$$\widehat{\psi_1}(\xi) = \frac{2\left(1 - \cos(\xi d)\right)}{\xi^2 d}, \xi \ne 0 \qquad (46)$$

Particular cases

(i) If we take $\tau_1 = \tau^1 = 0,\ \varepsilon = 1$, in Eqs. (38)- (42), we obtain the corresponding expressions of stresses, displacements and temperature distribution for micro stretch thermo elastic half space with one relaxation time.

(ii) If we take $\varepsilon = 0$ in Eqs. (38)- (42), the corresponding expressions of stresses, displacements and temperature distribution are obtained for micro stretch thermo elastic half space with two relaxation times.

(iii) Taking $\tau^0 = \tau^1 = \tau_0 = \tau_1 = 0$ in Eqs. (38)- (42), yield the corresponding expressions of stresses, displacements and temperature distribution for micro stretch coupled thermo elastic half space.

Special case

Micropolar Thermoelastic Solid: In absence of mass diffusion effect in Equations (38)- (42), we obtain the corresponding expressions of stresses, displacements and temperature for micropolar generalized thermoelastic half space.

Inversion of the transforms

The transformed displacements, stresses and temperature changes are functions of the parameters of Laplace and Fourier transforms s and ξ respectively and hence these are of the form $f(s, \xi, z)$. To obtain the solution of the problem in the physical domain, we must invert the Laplace and Fourier transform by using the method applied by Kumar (2005).

VI.NUMERICAL RESULTS AND DISCUSSIONS

The analysis is conducted for a magnesium crystal-like material. For numerical computations, following Eringen (1999), the values of physical constants are:

$\lambda = 9.4 \times 10^{10}\text{Nm}^{-2}, \mu = 4.0 \times 10^{10}\text{Nm}^{-2}, K = 1.0 \times$

$10^{16}\mathrm{Nm}^{-2}, \rho = 1.74 \times 10^{3}\mathrm{Kgm}^{-3}, j = 0.2 \times 10^{-19}\mathrm{m}^{2}, \quad \gamma = 0.779 \times 10^{-9}\mathrm{N}$,

Following Dhaliwal (1983) thermal and diffusion parameters are given by:

$$c^{*} = 1.04 \times 10^{3}\mathrm{JKg}^{-1}\mathrm{K}^{-1}, \qquad K^{*} = 1.7 \times 10^{6}\mathrm{Jm}^{-1}\mathrm{s}^{-1}\mathrm{K}^{-1},$$
$$\alpha_{t1} = 2.33 \times 10^{-5}\mathrm{K}^{-1}, \alpha_{c1} = 2.48 \times 10^{10}\mathrm{K}^{-1}, \quad T_{0} = 298\mathrm{K}, \quad \tau_{0} = 0.02, \quad \tau_{1} = 0.01, \alpha_{c1} = 2.65 \times 10^{-4}\mathrm{m}^{3}\mathrm{Kg}^{-1},$$
$$a = 2.9 \times 10^{4}\mathrm{m}^{2}\mathrm{s}^{-2}\mathrm{K}^{-1}, b = 32 \times 10^{5}\mathrm{Kg}^{-1}\mathrm{m}^{5}\mathrm{s}^{-2}, \quad \tau^{1} = 0.04, \quad \tau^{0} = 0.03, \quad D = 0.85 \times 10^{-8}\mathrm{Kgm}^{-3}\mathrm{s}$$

A comparison of the dimensionless form of the field variables for the cases of micro polar mass diffusion thermo elastic medium (MPMD) and micro polar thermo elastic medium (MP) for two different values of time *t* (*t=.01* and *t=.02*), subjected to linearly distributed source is shown in Figures 1-5. The values of all physical quantities for all cases are shown in the range $0 \leq x_1 \leq 2$.

Solid lines, dash lines corresponds to micro polar thermo elastic mass diffusion medium (MPMDT1) for *t =0.01*andmicropolar thermo elastic mass diffusion medium (MPMDT2) for*t=0.02* respectively.

Solid lines with central symbol & dash line with central symbol corresponds to micro polar thermo elastic (MPT1 and MPT2) for *t=.01* and *t=.02* respectively.

Linearly distributed normal force:

Fig. 1 shows the variation of normal stress t_{33} with the distance x_1. It is noticed that for MPMDT1 and MPMDT2, t_{33} show similar behavior. The value of normal stress monotonically increases as x_1 and then oscillates. The value of t_{33} increases near the application of the normal force due to the mass diffusion effect and then remain oscillating for all values of x_1.

Fig. 2 displays the variation of tangential stress t_{31} with the distancex_1. It is noticed that initially the behavior of t_{31} for MPMDT1 and MPT1 show variable trend but for MPMDT1, MPMDT2 and MPT1, MPT2 exhibits similar behavior. t_{31} Initially decrease monotonically for all the cases. The variation in tangential stress in micropolar thermoelastic is more than that of micropolar thermoelastic with mass diffusion.

Fig. 3 shows the variation of couple tangential stress m_{32} with distancex_1 . The behavior and variation of m_{32} for MPMDT1, MPMDT2, MPT1 and MPT2 remain similar to each other for all values of x_1. The magnitude of couple tangential stress in micropolar thermoelastic with mass diffusion is more than that of micropolar thermoelastic

Fig. 4 depicts the variation of temperature T with distance x_1. The trend and variation of T is similar in case of MPMDT1, MPMDT2 and MPT1 initially. For these curves the initial behavior is monotonically decreasing and oscillator away from the point of application of normal force. MPT2 show opposite trend initially.

Fig. 5 displays the variation of mass concentration C with distancex_1. For MPMDT1 and MPMDT2 the graphs are similar. Initially the trend is decreasing. After some oscillatory behavior mass concentration approaches to the boundary surface away from the application of force.

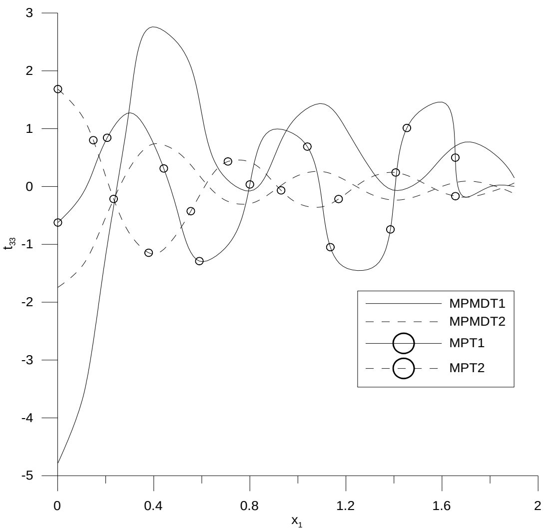

Fig. 1. Variation of normal stress with x_1

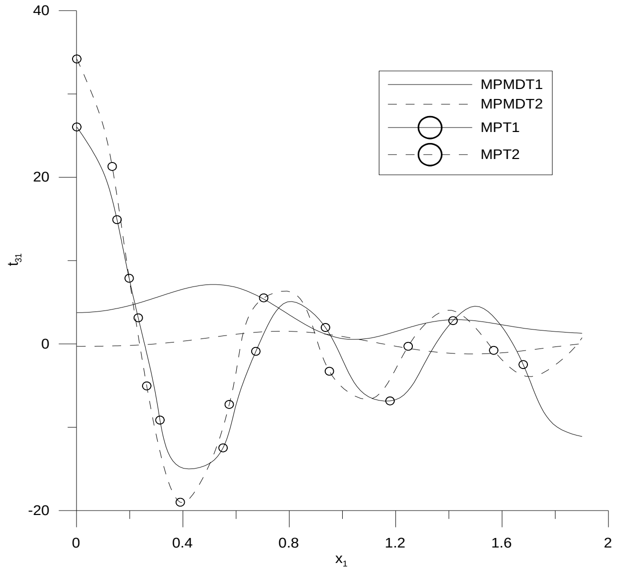

Fig. 2. Variation of tangential stress with x_1

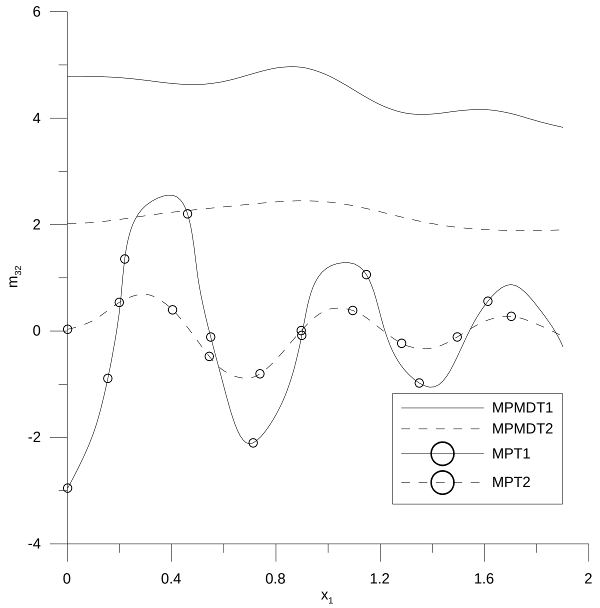

Fig. 3. Variation of couple tangential stress with x_1

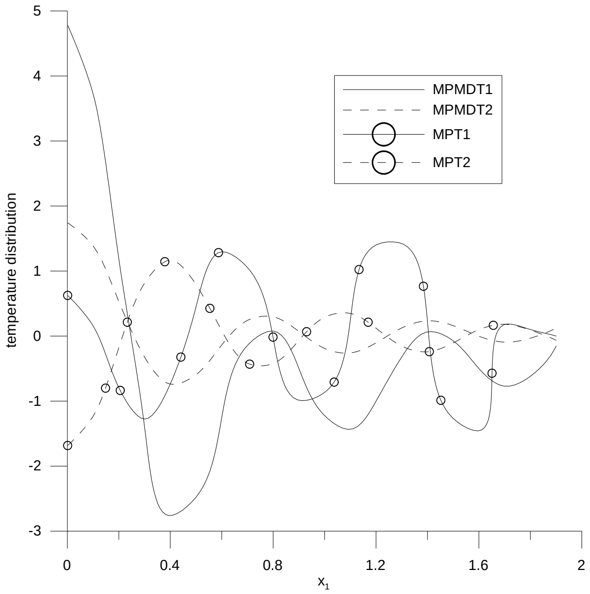

Fig. 4. Variation of temperature distribution with x_1

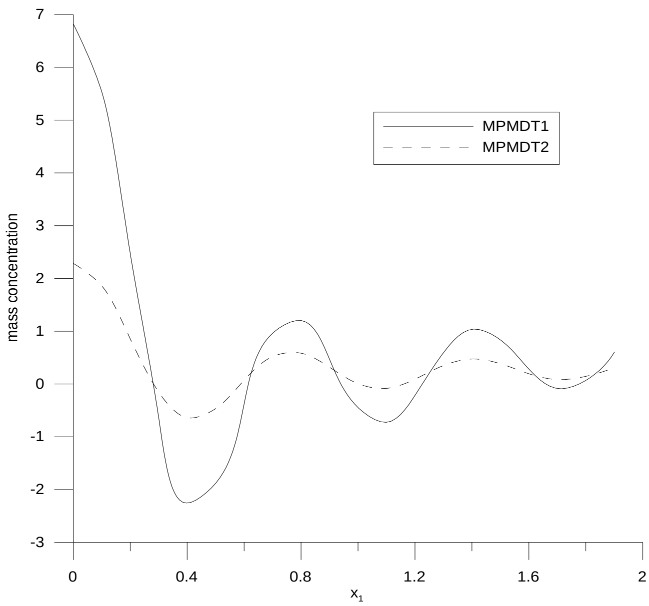

Fig. 5. Variation of mass concentration with x_1

VII.CONCLUSION

The problem consists of investigating displacement components, scalar mass concentration, temperature distribution and stress components in a homogeneous isotropic micropolar mass diffusion thermoelastic half space due to various sources subjected to laser pulse. Integral transform technique is employed to express the results mathematically.Theoretically obtained field variables are also exemplified through a specific model to present the results in the transformed domain.

The analysis of results permits some concluding remarks:

(1) It is clear from the figures that all the field variables have nonzero values only in the bounded region of space indicating that all the results are in agreement with the generalized theory of thermo elasticity.

(2) The effect of the mass concentration is much pronounced in all the resulting quantities.

(3) It is noticed that the figures that the time t plays a significant role in all the field quantities. Changes inthe value of time t cause significant changes in all the simulated resulting quantities.

(4) It can be easily concluded from the figures that the curves for various stresses in case of micro polar mass diffusion solid show similar trends.

(5) The variation of mass concentration differs significantly due to the presence of normal force and due to the presence of thermal source.

(6) Tangential stress, couple stress and temperature change are also affected due to diffusion effect as well as load/source applied.

The new model is employed in a micro polar mass diffusion thermo elastic medium as a new improvement in the field of thermo elasticity. The subject becomes more interesting due to irradiation of a laser pulse with an extensive short duration or a very high heat flux has found numerous applications. The method used in this article is applicable to a wide range of problems in thermodynamics. By the obtained results, it is expected that the present model of equations will serve as more realistic and will provide motivation to investigate micro stretch generalized thermo elasticity problems regarding laser pulse heat with high heat flux and/or short time duration.

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