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MODAL ANALYSIS OF COUPLED STRUCTURES AND PARAMETRIC RELATION OF THE COUPLED AND UNCOUPLED NATURAL FREQUENCY OF CYLINDERS BY FINITE ELEMENT ANALYSIS

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Abstract The purpose of this investigation is to explore the interaction of fluids with thin cylindrical structures of finite length. Our emphasis is on describing or approximating the coupled natural frequency of the fluid-filled structures as a function of modes of the uncoupled structures. This paper reviews the historical and theoretical findings of the relevant phenomenon important to the structural acoustics, modal frequencies of the cylindrical shell structures. The study focuses on the parametric analysis of fluid coupled thin cylindrical structures by Finite Element Analysis. Our specific contribution is defining a factor for analyzing and predicting the natural frequency of the water coupled cylinders for a given empty cylinder. Proposed a condition to differentiate the strong and weak coupling of the fluid and structure from a structural analysis point of view.

Key words: cylindrical structure, coupled, uncouple, coupling factor

I. INTRODUCTION

The purpose of this investigation is to explore the interaction of fluids with thin cylindrical structures of finite length. Our emphasis is on describing or approximating the coupled natural frequency of the fluid-filled structures as a function of modes of the uncoupled structures. This paper reviews the historical and theoretical findings of the relevant phenomenon important to the structural acoustics, modal frequencies of the cylindrical shell structures. The study focuses on the parametric analysis of fluid coupled thin cylindrical structures by Finite Element Analysis. Our specific contribution is defining a factor for analyzing and predicting the first few natural frequencies of the water coupled cylinders for a given empty cylinder. Proposed a condition to differentiate the strong and weak coupling of the fluid and structure from a structural analysis point of view.

II. THEORY INTRODUCTION AND THEORETICAL BACKGROUND

Acoustical vibration of pipes has undoubtedly been of interest since the earliest pipe installations. A structure vibrating in contact with the fluid of comparable density experiences loading proportional to the fluid inertial and elastic forces. Fluid loading thus modifies the forces acting on the structure and, since these acoustic pressures depend on the velocity, a feedback coupling between the fluid and structure exists. Hence the structural and acoustical domains must be solved simultaneously. The two domains are said to be coupled when boundary conditions ensure the continuity in displacement and pressure normal to the surface of the structure and fluid at the interface. Study on vibroacoustic (Structural Acoustics) on pipes and cylinders dates to the late 19th and early 20th centuries with the work of Rayleigh, Lamb, Love, and Stokes with the mathematical descriptions. Beginning in 1966, Ram Kumar and others (*10*) analyzed cylinder vibration and wave propagation in fluid-filled cylinders using the exact linear elastic theory, and both axisymmetric and symmetric vibration involving bending and flexural modes are discussed in these papers. The main emphasis of this paper was the calculation of the propagation speeds for each mode as a function of frequency. 1972 Miguel C. Junger (*1*) discussed intensively the integrated equation of structure-fluid interaction. He formulated the equations for normal modes of general structures, coupled with fluids. The discussion in this is restricted to specific boundary conditions and modes of vibration. Fluid-structure interaction has been formulated at different levels of complexity of physical representative. From the classical Westergaard, or 'added mass,' approach in which fluid incomprehensibility and rigid structure are assumed to the modern fluid-structure coupled boundary interface approach using the finite element analysis software. It is necessary to study the fluid-structure interaction problems in a coupled manner considering the flexibility effect of the structure and compressibility of the fluid. Formulations based on displacement variables are generally chosen for the structure, while the fluid is described by different variables such as displacement, pressure, velocity potential, etc. for such coupled problems. The recent (2014) investigation on the interaction of acoustics and vibration in fluid-filled cylindrical structures is done by David Chamber (*3*). He emphasis on describing longitudinal (axial) propagation within the structure for acoustic signals that enter externally. He also discusses on how dense fluids plays a significant role in structural vibration. Now with the increased capability of computers, it is permitting the analysis of more complicated vibration problems using Finite Element Method. With the access to the commercial software, solver database, one can get the detailed equations of the shape functions and matrix used in the solver and the principle behind it.

Finite element formulation (*5-9*) of the governing equations including the coupling conditions yields that the system of equation of motion for an undamped structure-acoustic problem can be written in the form.

$$\begin{bmatrix} M_s & 0 \\ \rho_0 c^2 H_{SF}^T & M_F \end{bmatrix} + \begin{bmatrix} \ddot{d}_S \\ \ddot{P}_F \end{bmatrix} + \begin{bmatrix} K_S & -H_{SF} \\ 0 & K_F \end{bmatrix} + \begin{bmatrix} d_S \\ P_F \end{bmatrix} = \begin{bmatrix} f_b \\ f_q \end{bmatrix} \tag{1}$$

Where;
d_S = Displacement variable in the structural
P_F = Nodal pressure
M_S, M_F = Mass matrix of the structure and fluid .
H_{SF} = Spatial coupling matrix
K_S, K_F = Stiffness matrix of structure and fluid.
ρ_0 = The static density of the fluid
c = Speed of sound in fluid.

The primary variables are the displacements in the structural domain and the acoustic pressure in the Fluid domain. It is evident that the two domains are inter-related. In fluid-filled shells, the mode structure can be understood as an interaction between the empty shell modes and the acoustic modes in the fluid. For the long or thick cylinders at the lowest frequencies, the shell modes are the simple combination of the empty shell modes (torsion, bending, extensional) and the plane wave acoustic mode. The fluid mass has a greater effect on the cylinders radial and bending mode, adding its inertia to the oscillations of the cross-section. Whereas for the thin cylinders when the radius to thickness ratio is large enough radial modes are more prominent than bending modes, which are normally observed at higher frequencies in long and thick cylinders. Perhaps the most important modes of pipe vibration are the flexural and bending modes. In these modes, the pipe radius varies with both angular and axial position. The coupling factor of the natural frequency can be linked to the variation of flexural wave velocity [Eq:2] (V_F) of the cylinder when its coupled with fluid (*1*). Different ways of finite element formulation of coupled structure-acoustic problems have been studied, starting from basic analysis by Ottosen and Petersson (*11*) to a more complicated dynamic response studied by Bathe (*12*).

In the present study, coupled free vibration of finite cylinders has been examined and tested by modeling and coding our implementations into the current commercial Finite Element Method software. The test program has been verified by comparison with the available numerical solutions in the theoretical work. The parametric study of the natural frequency has been examined in various combinations of geometries, materials and boundary conditions of thin-walled structures and fluids. The systems studied here are limited to the closed cylinder that consists of an acoustic fluid(water) which is coupled to the flexible structure. Our description of cylinder vibration ignores any effect of the external fluid to the pipe. In many applications, the effect is minor (as the outer fluid is air) and can be ignored until the analysis is of underwater acoustics or of very higher modes and frequencies.

$$V_F \cong \sim\omega/\mathrm{k} \qquad (2)$$

Where;
$k =$ Wave number of the strucuture
$\omega =$ Circular Frequency

III. PROBLEM STATEMENT

In the paper published by Amabili et.al., (*14*) the effect of fluid to enhance the non-linear behavior of the shell vibration, and the importance of non-linear analysis in fluid-filled shells than the empty one is discussed in detail. In the normal engineering field, such an analysis for all the coupled systems is cumbersome even with the help of finite element method (FEM) analysis. An important problem encountered in the structure-acoustic analysis using FEM is that the number of degrees of freedom easily becomes very large when solid element 'SOLID-186' are used to represent structures to achieve symmetrically coupled matrix formulation with the fluid element 'FLUID-220'. Another problem connected with fully symmetrical coupled matrix formulation is the incompatibility of shell elements in modeling the symmetric matrix of coupled motion of thin-wall and fluid. But 'SHELL-63' a structural shell element will couple with fluid element FLUID-30 with the lack of symmetry in the system of equations and coupling matrix. In all these methods the large bandwidth of the system matrices and the coupling matrix add up to long computational time.

it is better to predict the coupling factor [Eq:20] in the structural design process rather than spending amount of significant time in monitoring the modal characteristics of every coupled structure(cylinder) if other dynamics properties of the fluids are not of practical interest. But predicting the natural frequency of the coupled structure is a complicated mathematical problem. The solution is only possible using a higher order equation, and theoretical solutions are restricted to specific models and boundary conditions. However, generalizing the prediction of coupled modal frequencies of the cylinders for all the boundary conditions and for all possible vibration modes in a complicated structure is still not practical. Hence for a specific familiar model and for a specific range of structural and fluid properties, a parametric study can be done to simplify the relationship between the coupled and uncoupled model to obtain an approximate solution to the first few modal frequencies of the coupled structure from the data of the uncoupled analysis. This saves a lot of computational time in the static structural design where the vibrational characteristics are not of primary objective but cannot be neglected in these specific cases. An attempt is made in this regard to achieve a feasible approximation of the first few natural frequencies of the thin cylinders under the influence of dense fluids as water, for a given natural frequency of these cylinders (empty).

3.1 Finite Element Formulation.

Several finite element formulations have been proposed for acoustic fluids in the analysis of fluid-structure interaction problems, namely, the displacement formulation, the displacement potential and pressure formulation, and the velocity potential formulation (Everstine, Olson and Bathe) (*13*). The displacement formulation has received considerable attention, because it does not require any special interface conditions or new solution strategies and because of the potential applicability to the solution of a broad range of problems. [Fig:1] is the classical modal behavior of the cylinders, where the natural frequencies increase with the mode number but first few natural frequencies correspond
to higher modes and first few modes occur at the higher frequencies. Hence there is an initial dip in the natural frequencies' v/s mode number curve. This dip is governed by the geometry of the cylinder under analysis. The details of the topic are explained in the sections [5,6,7] later. In the Finite element formulation, a system of equations describing the motion of the system is developed (*2, 5–9*), with the number of equations equal to the number of degrees of freedom introduced in the Finite Element discretization. For the structure-acoustic system, the structure is described by the differential equation of motion for a continuum body assuming small deformations and the fluid by the acoustic wave equation. Coupling conditions at the boundary between the structural and fluid domains ensure the continuity in displacement and pressure between the domains.

The governing equations of each domain based on their boundary conditions are:

$$\text{Structure}: \widetilde{\nabla}^T \sigma_S + b_S = \rho_S \frac{\delta^2 u_S}{\delta t^2}$$
$$\text{Fluid}: \frac{\delta^2 p_F}{\delta^2 t} - c^2 \nabla^2 p_F = c^2 \frac{\delta q_F}{\delta \mathrm{t}} \qquad (3)$$
$$\text{Coupling}: \overrightarrow{\overline{\sigma_S}\vec{n}} + \mathrm{p}\vec{n} = \vec{n}\cdot\overrightarrow{u_S} - \vec{n}\cdot\overrightarrow{u_F} = 0$$

Where;
$u_S, u_F =$ Displacement of Solid and Acoustic Fluid;

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p_F = Pressure Field in the Acoustic Domain;
b_S = Body Force
σ_S = Solid Stress Tensor
n = Outward Normal Unit Vector of Fluid Domain
q_S = Inertia Force

$$\widetilde{\nabla}^T = \begin{bmatrix} \delta/\delta x1 & 0 & 0 \\ 0 & \delta/\delta x2 & 0 \\ 0 & 0 & \delta/\delta x3 \\ \delta/\delta x2 & \delta/\delta x1 & 0 \\ \delta/\delta x3 & 0 & \delta/\delta x1 \\ 0 & \delta/\delta x3 & \delta/\delta x2 \end{bmatrix}$$

To arrive at the finite element formulation for these individual domains, a weak form of the differential equations is derived *[2, 7-11]*. The Finite Element Formulation is formed by using the integral of the Governing equation with the structural domain. Without going into much detail derivation of the above function, using shape function, displacement domain and weight function, the Finite Element structural domain can be formulated as in,

$$\int N_S^T \rho_S N_S dV \ddot{d_S} + \int \left(\widetilde{\nabla} N_S\right)^T D_{S\widetilde{\nabla}} N_S \, dV d_S = \int N_S^T t_S dS + \int N_S^T b_S dV \qquad (4)$$

and this governing system of equations can be written as;

$$M_{S\ddot{d_S}} + K_S d_S = f_F + f_b \qquad (5)$$

Where;

$M_S = \int N_S^T \rho_S N_S dV$;

$K_S = \int \left(\widetilde{\nabla} N_S\right)^T D_{S\widetilde{\nabla}} N_S \ dV$

$f_F = \int N_S^T t_S d_S$

$f_b = \int N_S^T b_S dV$

N_S = Finite Element Shape Function of Structure
d_S = Displacement Matrix
D_S = Constitutive Matrix
b_S = Inertia Matrix
t_S = Surface Traction Vector

Similarly considering the governing equation of an in-viscid acoustic fluid a weak Finite Element formulation can be done by integrating the time derivative of governing equation of fluid over volume (*5–9*). Expressing the pressure field of the acoustic fluid as the shape function and weight function, a Finite Element of governing equation can be formed for the fluid domain.

$$\int N_F^T N_F dV \ddot{p_F} \, c^2 + \int \left(\widetilde{\nabla} N_F\right)^T \widetilde{\nabla} N_F dV p_F c^2 = \int N_F^T \nabla p_F n_F dS \, c^2 + \int N_F^T \frac{\delta q_F}{\delta t} d_V$$

and this governing equation can be written as;

$$M_{F\ddot{p}} + K_F P = f_s + f_q \qquad (6)$$

Where;

$M_F = \int N_F^T N_F dV$;

$K_F = c^2 \int \left(\widetilde{\nabla} N_F\right)^T \widetilde{\nabla} N_F dV$

$f_S = c^2 \int N_S^T t_S d_S$;

$f_q = c^2 \int N_F^T \nabla p_F n_F dS$

N_F = Finite Element Shape Function of Acoustic
p_F = Pressure field of the fluid
n_f = Vector normal of the fluid

Now the Coupled Structure-Acoustic System can be defined at the interface of the fluid and structure. At the boundary, the fluid and structural nodes should have same displacement and pressure in the normal direction. Hence, the displacement and pressure boundary conditions can be written as,

$$\overrightarrow{\overline{\sigma_S}}\vec{n} + \mathrm{p}\vec{n} = 0 \qquad (7)$$
$$\vec{n} \cdot \overrightarrow{u_S} - \vec{n} \cdot \overrightarrow{u_F} = 0$$

Now the coupling can be introduced in the form of force f_S and f_F by relating pressure and the acceleration of the fluid and structural domain at the boundary. Structural force f_F is related to both the stress and pressure acting on the structure and fluid as.

$$f_F = \int N_S^T n N_F \, d_S P_F \qquad (8)$$

Similarly, the force acting on the fluid f_S is related to the acoustic fluid pressure and stress.

$$f_S = -\rho_0 c^2 \int N_F^T n^T N_S d_S \ddot{d}_S \qquad (9)$$

Where:
$p_F = N_F P_F$
p_F = Pressure Field
P_F = Nodal Pressure
N_F = Finite Element Shape Function of Fluid .

The structure-acoustic problem in the Finite Element method as expressed in Eq 3, the HSF can be defined as a spatial coupling matrix:

$$H_S \mathrm{F} = \int N_S^T n N_F \, \mathrm{dS} \qquad (10)$$

IV. VALIDATION OF FINITE ELEMENT METHOD USED FOR ANALYSIS

In order to examine the feasibility and the accuracy of the proposed iterative scheme, a benchmark problem has been solved and compared with the existing literature. In the book 'Sound Structure and their Vibration' by Miguel C. Junger (*1*), there is an extensive discussion about the coupling of structure and fluids at a different range of frequencies. The characteristic equation of the fluid filled sphere is given by:

$$\frac{\rho_S c_p^2 h}{3\rho c^2 a}(\Omega^2 - \Omega_0^2) = \frac{k_0 a}{3\rho c^2} \qquad (11)$$

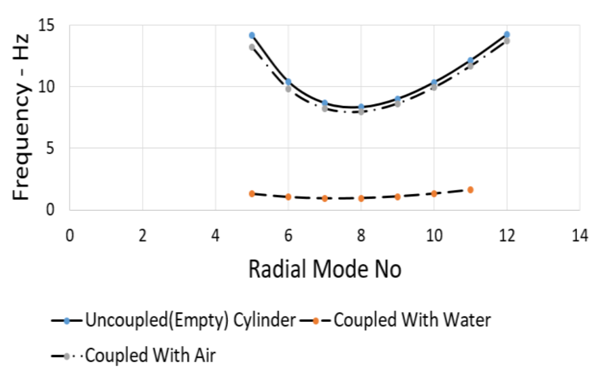
where;
c_p = Phase velocity of the compressible wave
c = Sound velocity
ρ_s = Density of the structural material
h = Shell thickness
ρ = Density of the fluid
a = Radius of the sphere
Ω = Dimensionless frequency of a vibrating sphere
$\Omega^2_0 = 2(1 + \nu)$
k_0 = Frequency dependent stiffness factor
ν = Poisson's ratio

This is a transcendental equation, because k_0 is frequency dependent. Thus, this equation has a series of solutions for a single mode.

4.1 Finite Element Method

Calculations were performed for the following structure and fluid properties:
Structure : Steel
Young's Modulus : $2.1e^{11}$Pa
Poisson's Ratio : 0.28
Fluid : Water
Density : 1000kg/m^3
Sonic Speed : 1484m/s
For a water-filled steel sphere of radius 1 m;
1^{st}breathingmode = 928Hz
2^{nd}breathingmode = 1653.18Hz
3^{rd}breathingmode = 2322.66Hz

Now in order to validate and verify the FEM model, the structure and fluid elements are coupled at the intersection. The numerical modeling is done using ANSYS- APDL 17.1. Two ways of FEM modeling is shown that represents the fluid-structure coupling: 3D Elements: The structures were modeled as a SOLID element-186 and Fluid was modeled as Acoustic Element-230. Shell Elements: The structures were modeled as a SHELL-63 elements and Fluid was modeled as Acoustic Element-30. Using LSIZE Command in ANSYS-APDL the circumference of the cylinder is divided into 40 segments and meshed accordingly to achieve the same number of elements per each half wavelength. So, the meshing is consistent with all the models used in the study.



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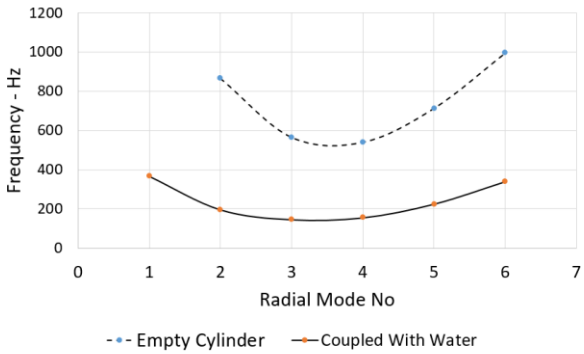

Figure-1. Natural Frequencies of Coupled and Uncoupled Cylinder

Breathing modes are those vibrational modes associated with a flexural motion of the shell wall such that the radial displacement is proportional to cos(n) where n=0 which corresponds to rotationally symmetric motion. Hence a frequency response analysis is done by applying equal pressure over the spherical surface. At the breathing mode frequency, there is least resistance to all the nodal displacements in the normal direction as a response to the symmetric pressure over the surface of a sphere. So, there will be a peak in the normal displacement of a given node at the breathing mode frequency and the corresponding normal displacement is plotted over the frequency for the outer surface nodes at x=0 and y=0 as observed in the graph Fig:[3, 4]

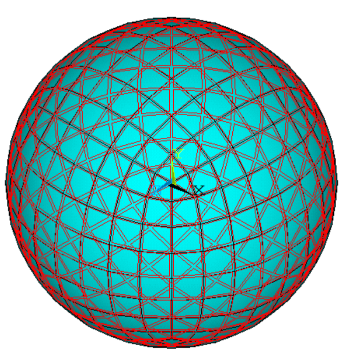

Figure-2. Sphere Mesh

The model [Fig:2] describes a sphere filled with fluid and the nodes on inner boundary of the sphere is coupled with the fluid elements inside. A pressure load is applied on over the surface of the sphere. As that pressure acts on the surface of the sphere and the frequency response of the sphere with respect to the normal displacement of the surface is plotted as shown in graphs [Fig:3,4]. There is significant normal displacement at a specific frequency which can be asserted as breathing mode of the sphere. More precise values of frequency were obtained by running the analysis in the close range of individual peaks.

Table 1: Breathing Modes of Coupled Sphere

Breathing Modes of Coupled Sphere			
No.	Theoretical Frequency (Hz)	Solid Structural Element (Hz)	Structure as Shell Element (Hz)
1	971.2	868	871
2	1653.7	1533.7	1543
3	2322.6	2230	2252

The FEM results are similar to those of theoretical results in Table [1]. The difference in

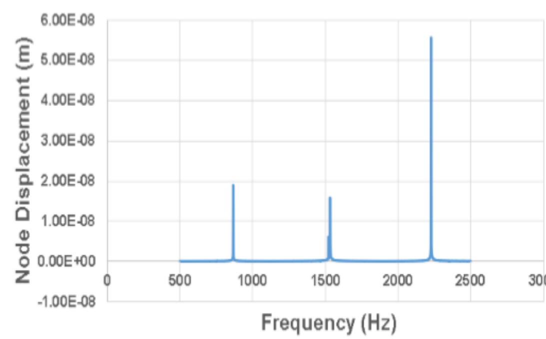

Figure 3: Frequency Response of Displacement of the Sphere - Solid Element

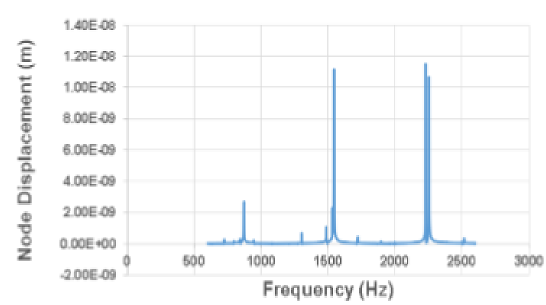

Figure 4: Frequency Response of Displacement of the Sphere - Shell Element

eigenvaluesislessthan5percent. The local vibrations and difference in the magnitude of nodal displacement is evident in shell element results, we are concerned about only the frequency of the breathing mode, other factors such as magnitude of the nodal displacement and local excitation will lead to another different study which is not our interest. Thus, results apparently convince that both FEM modeling method is right. It is also noted that for uncoupled sphere all the models provide 1400Hz as the breathing mode. On completing the analysis by using both symmetrical and unsymmetrical coupled matrix, it is evident that the accuracy of both the methods are agreeable given that the shell elements does require a dense mesh to avoid the numerical errors in the plots. For the given number of elements and mesh density, the symmetric matrix does solve it more efficiently, but a lot of care must be taken in meshing the structure. Shell elements are used for this parametric study as numerous interactions are done by changing the model parameters for the analysis.

V. THEORY ON COUPLED CYLINDERS

Let's consider a simple elastic shell structure enclosing the fluid. The cylinder wall is assumed to be locally reacting and therefore characterized by a specific acoustic reactance, x_w.

For a cylinder of radius $'a'$ the radial velocity $\dot{\omega}$ is related to the pressure.

$$\dot{\omega} = -\mathrm{i}\frac{(\delta p/\delta r)}{\rho c k} \qquad (12)$$

Boundary condition for the interface can explicitly state as,

$$\frac{p}{(\delta p/\delta r)} = \frac{x_W}{\rho c k} \qquad (13)$$

For air-filled cylinder, the wall can be approximated as rigid. For a water-filled cylinder, the elastic property of the cylinder is not negligible and influences wave propagation. Neglecting longitudinal pipe vibration, an axisymmetric sound pressure produces a uniform radial deflection, which results in a hoop strain and stress.

$$\epsilon = \frac{\omega}{a} \qquad (14)$$

$$\sigma = \epsilon\frac{E}{1-\nu} \qquad (15)$$

Interaction solutions are usually more readily interpreted when the structural elastic constants are expressed in terms of concepts related to the wave propagation. Hence a phase velocity in a structure is introduced. For a plate, the phase velocity is given by [Eq:16],

$$c_P = \mathrm{E}/[(1-\nu^2)\rho_s]^{1/2} \qquad (16)$$

Fractional volume change is enhanced by the compressibility of the acoustic fluid and wall elasticity. Thus, the sound velocity of the fluid is changed, which is given as a correction factor in the equation below.

$$B_W^{-1} = (dV/V)/\mathrm{p} = 2\omega/\mathrm{ap} = 2\mathrm{a}/\rho_s c_p^2 \mathrm{h}$$

Where,
B = Bulk modulus of the fluid $= \rho\, c^2$
$\mathrm{B_w}$ = Bulk modulus as the function of sound velocity in the fluid

The sound velocity can now be corrected for wall elasticity by noting that the compressibility of the acoustic fluid and of the pipe wall combine so as to enhance the fractional volume change:

$$c_0 = [\rho(B_W^{-1}/B^{-1})]^{-1/2} \qquad (17)$$

c_0 = Phase velocity of the fluid filled cylinder
As a comparison or a correction factor,

$$c_0/\mathrm{c} = [1 + (B_W^{-1}/B^{-1})]^{-1/2} \qquad (18)$$

Substituting the Bulk modulus of the fluid this becomes;

$$c_0/\mathrm{c} = \left[1 + 2a\rho c^2/\left(h\rho_s c_p^2\right)\right]^{-1/2} \qquad (19)$$

This equation is called as 'Korteweg-Lamb' (1) Correction. In a similar approach, taking the cylindrical modes into considerations, this paper is defining a correction factor for predicting the natural frequency of the coupled system for the given uncoupled natural frequency of the cylinder of the same mode.

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VI. MODAL CHARACTERISTICS OF CYLINDERS COUPLED WITH FLUIDS

In this section, an attempt is made to solve the fluid coupled structural vibration response. As stated previously the modeling method is proved right. The structure is represented by SHELL-63 Elements and fluid by Acoustics-30 element.

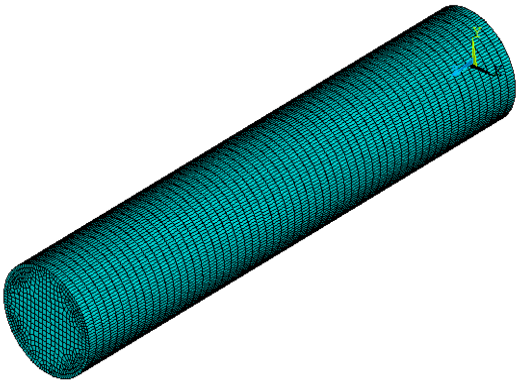

Figure 5. Cylinder Coupled Mesh

Figure 4. shows the mesh of closed cylinder with 2m radius and 5m length with 1mm thickness containing fluid. The modal behavior of these structures is investigated by comparing with coupled and uncoupled condition.

The cylinder is designed as a thin and short cylinder to avoid more longitudinal or bending modes as the first 20 eigenvalues. The circumferential modes are of interest in this study, the reason for this is explained in further discussions.

$$\text{Coupling Factor} \cong \frac{\text{Natural Frequency of Coupled}}{\text{Natural Frequency of Uncoupled}} \tag{20}$$

The importance of coupling factor can be seen in the Tables [3,4] where for the empty cylinder the first natural frequency is 25Hz at which the cylinder undergoes a large displacement with the least resistance. But when it is coupled with water same state of vibration occur at 6Hz. So, it should be noted that if a cylinder is installed in the dynamic system under the above stated condition, care must be taken to damp this mode which is over the range of 6Hz to 25Hz.

The analysis is done for cylinders with following Boundary Conditions:

1) Fixed-Fixed boundary - Where both ends of the cylinder are fixed in all the directions.

2) Fixed-Free boundary - Cantilever cylinder i.e. one end is fixed, and the other end is free.

Table [3] represents the natural frequency of the uncoupled cylinder-steel and the coupled condition with fluids, air and water respectively. It is evident that the coupling factor of the structure with air is negligible compared to that of water. It should be noted that the radial mode number and modal frequency are related as shown in the graph [Fig:7]. The curves show that the first few modes are not tabulated as they occur at a very higher frequency, those modes are out of interest for this study as we are concern about the first few natural frequencies. There is still discussion on defining coupling factor as weak coupling and strong coupling, but in this case, it can be asserted that the air has a very weak coupling factor with respect to this structure, on the other hand, water can be treated as strong coupling. Moving forward with the assertion the Table [4] and Graph [Fig:7] is representing the coupling factor of the same model with the different structural property - Aluminum. Comparing the results of these two cylinders, it is obvious that for a given fluid the coupling can be significant or insignificant based on the material property of the structure, so a soft structure such as polymers, could have a coupling factor with high-pressure air which can't be neglected. Hence our area of interest is to deal with these strong coupling. However, the comparison of the empty and coupled natural frequency of the structure is complicated as explained in the previous chapter. The relationship between these frequencies is of higher order differential equations and complex. An attempt is made to relate these frequencies to a dimensions, properties and boundary conditions of the cylinder and fluid. A parametric study is performed for each set of data input by modeling numerous models of varying factors, which are explained in detail in the coming chapters.

Table 1. Mode Shapes

Radial Modes: m=1

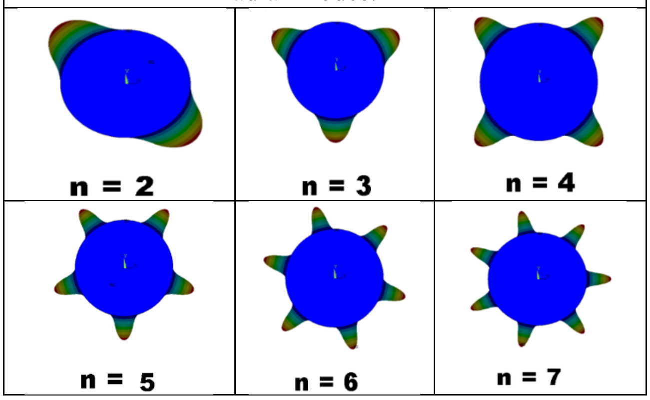

Table 2 : Radial modes and natural frequency of couple and uncouple cylinder (Hz)

Radial modes and natural frequency of couple and uncouple cylinder (Hz)			
Radial	Empty	Coupled	Coupled

Mode No.	Cylinder	with air	with water
3	36.797	36.388	8.343
4	25.665	25.442	6.5202
5	26.993	26.802	7.537
6	35.234	35.022	10.645
7	47.083	46.839	15.220

Table 3 : Coupling factor of steel and aluminum coupled with air and water

Coupling factor of steel and aluminum coupled with air and water				
Radial mode No.	Aluminum		Steel	
	Air	Water	Air	Water
5	0.837	0.053	0.934	0.091
6	0.857	0.058	0.943	0.099
7	0.873	0.062	0.949	0.107
8	0.886	0.066	0.953	0.114

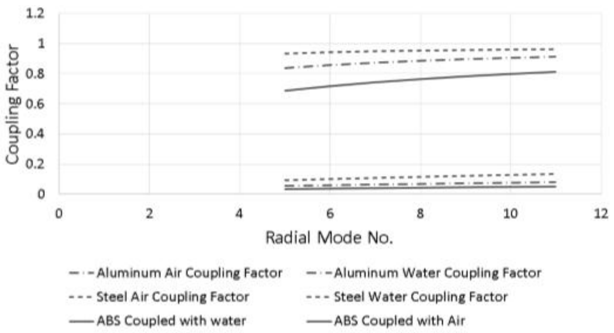

Figure 6.

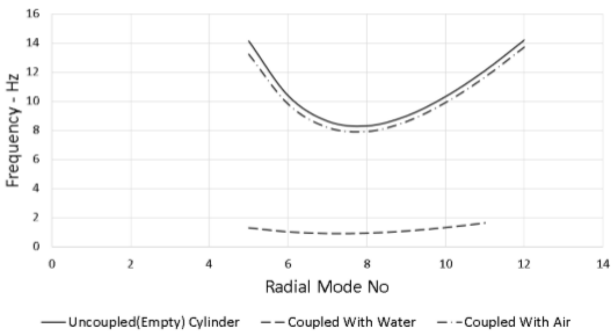

Figure 7.

VII. PARAMETRIC STUDY OF THE MODAL CHARACTERISTICS OF ﬂUID COUPLED CYLINDERS:

To study the effect of the structural and fluid properties and the geometric parameter of the cylinder on the coupling, individual parameters including the thickness, radius, length, of the cylinder and density of the structure and fluid, are to be analyzed independently by keeping the other parameters constant. The specific parameter must be isolated to analyze the effect and significance of that parameter in the engineering design. The effects of these parameters at a different span of geometry and materials are studied. Furthermore, the fluid properties contribute significantly to the coupling and hence drive the coupling factor which we are attempting to predict. As described in the problem statement the importance and the non-linear behavior with the fluid in modal analysis of the cylinder, it is not practical to formulate a prediction which can be generalized to all fluids and structural conditions. Hence all the parametric study is confined to one fluid at a time to achieve more precise variations and dependencies of the other parameters over the coupling of the structure with the selected fluid. The fluid selected for the study in this paper is water. All the analysis carried out is with regarding the coupling characteristics of water with cylindrical structures.

7.1 Parametric study on relation between modal frequencies of uncoupled, and the cylinders coupled with water

Many Finite Element Models were constructed, and normalizing techniques were used to help generalize the results for a specific range of models. For all models, the first 20 to 40 natural frequencies of the systems are calculated with the geometrical and structural properties mentioned in the Table:[5]. In this analysis, some metal and polymer cylinders are examined. The cylinder dimensions are varied to get the desired mode shapes for the analysis on the effect of these variables on the coupling. The following description summarizes the effects that these parameters have on coupled natural frequencies of the cylinder.

7.1.1 Effect of thickness and radius of cylinder on coupling

In this section, 2 geometrical parameters are considered as the design variables affecting the coupling factor. To show the effect of thickness and radius on coupling factor, a series of models were developed by considering a non-dimensional geometric factor defined as the ratio of radius to thickness of the cylinder. To obtain a clear variation with respect to thickness and radius in metal structures, an ideal model is constructed with very thin cylinder ranging from a thickness of 0.1mm to a thicker model of 5mm and large radius of 0.5m. So, the dimensionless geometric ratio varies from 5000 to 50. Anyhow for a soft polymer material like Acrylonitrile Butadiene Styrene (ABS), the thickness ranges from 1mm to 5mm and radius from 10cm to 50cm, here the ratio varies from 100 to 20. Figure [8,9,10] shows the relationship between the geometric factor and the change in coupling factor of the system with different materials

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and radial mode number for both fixed ends and cantilever boundary conditions.

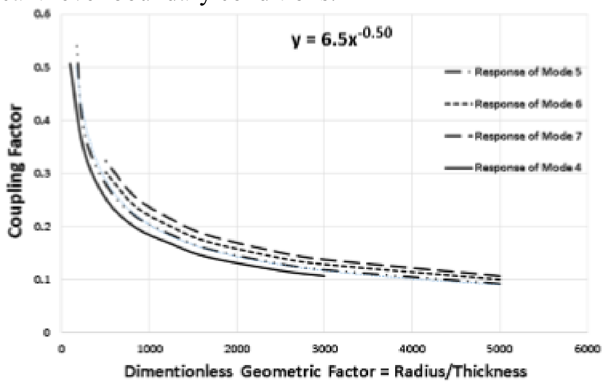

Figure 8. Effect of r/t on coupling factor in steel cylinder with fixed end BC

The model used in the graph [Fig:8] is the steel cylinder of 5m length coupled with water. It is shown that the coupling factor decreases with the increase of the radius to thickness ratio. It should be noted that the effect of the coupling is stronger because, ABS is soft material and it is not dense and stiff enough compared to metals to resist the interaction with fluid motion. Thus, even thicker ABS cylinders couple with the fluid vibration at much lower frequency compared to metals and will have a stronger coupling. Indeed, the variation for all the modes are same. The relation is shown in the graph [Fig:8] as a power of−0.5. Similarly, another modal is built using ABS material and coupled with water. The relationship is shown in the graph [Fig:9]. Also, with the change in material property and boundary condition, the variation of coupling factor w.r.t geometric parameter is almost unchanged [Fig10]. Therefore, the coupling factor is inversely proportional to the square root of the non-dimensional geometric ratio.

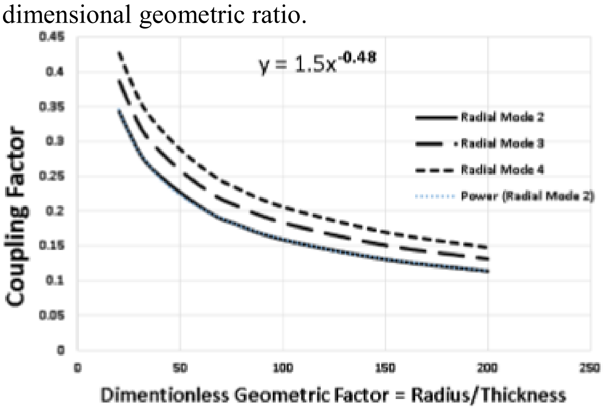

Figure 9. Effect of r/t on coupling factor in ABS cylinder with fixed end BC

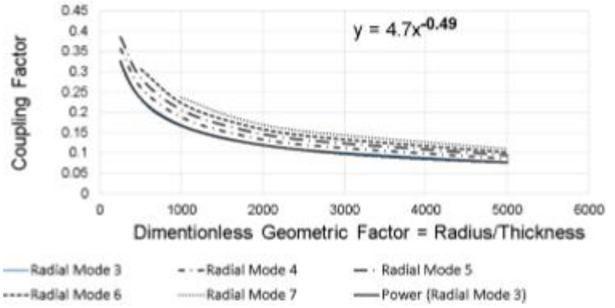

Figure 10. Effect of r/t on coupling factor in steel cylinder with cantilever end BC

$$\text{Coupling Factor} \propto \sqrt{\frac{1}{r/t}} \propto \sqrt{\frac{t}{r}} \tag{21}$$

7.1.2 **Effect of Structural material density on coupling**

To investigate the effect of structural material density on coupling factor considering the properties of the material, models of the coupled cylinders were created with different materials with varying density. The coupling factor is calculated for same modes at a time. Graph [Fig:11] shows the effect of structural density on the natural frequencies (coupling factor) of the water coupled cylinder with the r/t ratio of 1000.

Of course, an increase in radial mode number leads to an increase in coupling frequency and vice versa, but it is evident that the variation of the coupling factor is also same for all the modes. Moreover, it is also obvious that the denser structural materials will have weak coupling effects and hence the coupling factor is less for the structures with the larger density, and the graph [Fig:11] indicated the variation is in the order of 0.5. Thus, it can be asserted that the coupling factor is proportional to the square root of the structural material density of the cylinder when coupled with water.

$$CouplingFactor \propto \sqrt{\rho} \tag{22}$$

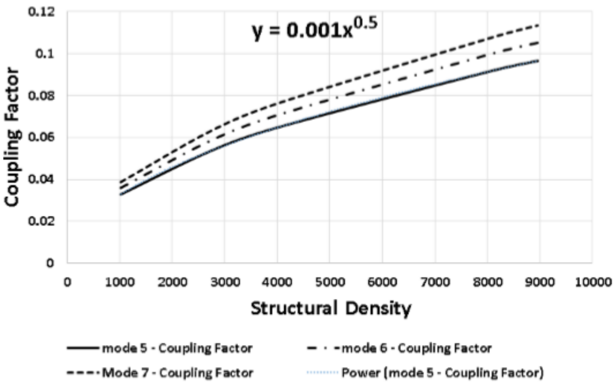

Figure 11. Effect of structural material density on coupling factor of water coupled cylinder with r/t=1000

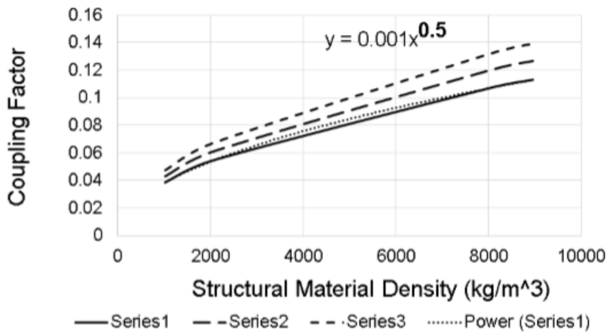

Figure 12. Effect of structural material density on coupling factor of water coupled cylinder with r/t = 2000 - Cantilever boundary condition

$$\frac{Natural\ Frequency\ of\ Coupled}{Natural\ Frequency\ of\ Uncoupled} = \sqrt{\frac{\rho_s nt}{\rho_w r}} \qquad (23)$$

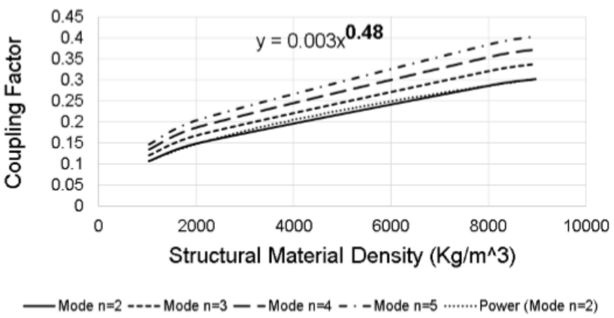

Figure 13. Effect of structural material density on coupling factor of water coupled cylinder with r/t = 250 - Cantilever boundary condition

7.1.3 **Effect of longitudinal modes of cylinder on coupling**

Throughout this study, importance is given only to the radial modes. The geometries of the cylinders are made so that longitudinal modes are avoided and first few natural modes for the convenience of the analysis. It is obvious to expect the least interdependency between these modes, as by definition of these different natural modes they are orthogonal to each other. Thus, there is no interrelation of these modes with respect to the coupling factor. To clarify this, Table: [6] compares the coupling factor of first longitudinal mode with higher longitudinal modes for the same radial mode number of the cylinders coupled with water. A similar analysis is done for numerous models to verify results. In all these analyses the effect of longitudinal modes of vibration on coupling factor is found to be very minimal or negligible.

Table 5 Coupling Factor for different longitudinal and radial modes

$4^{th} Radial\ Mode$		$5^{th} Radial\ Mode$		$6^{th} Radial Mode$	
Longitudinal Mode No.	Coupling Factor	Longitudinal Mode No.	Coupling Factor	Longitudinal Mode No.	Coupling Factor
1	0.170	1	0.186	1	0.203
2	0.170	2	0.187	2	0.203
3	0.171	3	0.188	3	0.204
-	-	4	0.189	4	0.205

7.1.4 **Expression to relate the natural frequency of the cylinder coupled with water and the empty cylinder.**

From the parametric study of all these above factors, the coupling factor which related the uncoupled to the coupled system can be expressed as a function of these variables in their respective orders.

Where;
ρs = Density of the structural material.
ρw = Density of water.
r = Radius of the cylinder.
n = Radial modes of cylinder.
t = Thickness of the cylinder shell.

This expression can be used for a wide range of structures and geometries of the cylinder with an error less than 5 percent. Results of all the analysis are not illustrated, but to get the sense of the breadth of the

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analysis, the results of variables with a significant difference in their magnitudes and ratios are tabulated below. For metal cylinders as of steel and aluminum whose stiffness and the density are much higher than that of the water, Tables: [7,8] show the FEM and results from the Eq [23] of the coupled and uncoupled modal frequencies. The expression will have least error only for very thin shells of the cylinder, but for soft materials as of ABS the calculated results match closely even for thick cylinders as illustrated in Table: [9,10].

Table 6: Approximated modal frequency of the coupled cylinder 1

Steel Cylinder with $r/t = 1000$ coupled with Water				
Radial Mode No.	Modal Frequency (Hz)			Error Percentage
	FEM Results		Approximated Results	
	Empty Cylinder	Coupled Cylinder	Coupled Cylinder	
9	8.901	0.769	0.757	1.47
10	7.481	0.680	0.671	1.36
11	6.568	0.628	0.618	1.55
12	6.062	0.604	0.595	1.47
13	5.893	0.613	0.602	1.8

As other factors are analyzed independently the variation in errors in these calculations are related significantly to the radial mode number. It is hard to track the nonlinear or higher order relations of radial modal frequency, with the coupling by a parametric study. So, the analysis is further constrained to a set of models for every structural material of the cylinder. Commenting on the results, it is a well-known fact that in the lower order natural frequency the fluid might act as an added mass which in turn impels that the density of the fluid and structure dictates the coupling provided that the structure is thin enough compared to its elasticity to lower the effect of stiffness of the structure so the error in the expression increases for thicker and small cylinders. It is obvious that the stiffness and the phase velocity of the structure and fluid plays a very important role in the coupling. As these are not considered explicitly in the expression, the effect of those parameters will eventually take precedence as the shells get thicker. This is also the reason for which the expression [eq: 23] is restricted to water and only for first few natural frequencies of the structures. The conditions where the eq: [23] is valid for the approximation are listed below for Steel (High density) and ABS (Low density - polymer);

Table 7: Approximated modal frequency of the coupled cylinder 2

Steel Cylinder with $r/t = 100$ coupled with Water				
Radial Mode No.	Modal Frequency (Hz)			Error Percentage
	FEM Results		Approximated Results	
	Empty Cylinder	Coupled Cylinder	Coupled Cylinder	
2	72.448	29.839	29.070	2.578
3	52.074	24.045	25.590	6.42
4	74.734	40.847	42.407	3.8
5	116.361	65.427	73.822	12.8
6	169.453	107.964	117.765	9.07

Table 8: Approximated modal frequency of the coupled cylinder 3

ABS Cylinder with $r/t = 200$ coupled with Water				
Radial Mode No.	Modal Frequency (Hz)			Error Percentage
	FEM Results		Approximated Results	
	Empty Cylinder	Coupled Cylinder	Coupled Cylinder	
3	23.22	3.02	2.88	4.9
4	18.53	2.72	2.66	2.4
5	22.49	3.65	3.60	1.23
6	30.90	5.45	5.43	0.48
7	41.83	7.94	7.94	0.01

For the cylinders lower than the stated radius to thickness ratio, the error in the approximation is no longer non-significant to neglect. The stiffness of the structure and thus the phase velocity need to be analyzed theoretically with higher order equations to get the relation with the coupling factor which is out of interest for this parametric study.

Table 9: Approximated modal frequency of the coupled cylinder 4

ABS Cylinder with $r/t = 20$ coupled with Water				
Radial Mode No.	Modal Frequency (Hz)			Error Percentage
	FEM Results		Approximated Results	
	Empty Cylinder	Coupled Cylinder	Coupled Cylinder	
2	167.98	57.64	53.91	6.46
3	166.31	64.31	65.37	1.64
4	267.11	114.01	121.23	6.33

$$Steel: \ 100 \leq r/t$$
$$ABS: \ 20 \leq r/t$$

VIII. CONCLUSION

Care should be taken while designing the cylinders, encountering dense fluids such that the geometry is feasible to avoid the strong coupling with the fluid. Flexural wave speed in the structure reduces due to the interaction with the dense fluid when compared to a given mode of vibration in vacuum [Eq:2]. Hence coupled structures achieve their first few natural modes at a lower frequency, and this theory is widely studied and accepted. So, the results obtained are convincing based on this theory. For the explanation that the variation of coupled natural frequency is very less dependent on the material stiffness (Modulus of elasticity) of the structure, it is convincing that the equation[23] is similar to the one discussed in an article by 'G.B. Warburton' (15) in which the formulated model is also independent of the elasticity modulus, except for the displacement component factor. But we can't move forward with that equation as its more generalized and equation is derived for the infinite shell. For a specific finite cylinder, it is equally complicated as of other theories with factors to be calculated based on the boundary conditions and displacement components. Whereas Eq: [23] is more specific and simple. The equation approximates facilitate the design in terms of selecting the appropriate materials for the required geometry or vice versa, irrespective of the boundary conditions. The Eq: [23] can be generalized in a way for wide variety of structures and fluids if similar models for proposed and tested in the future studies;

$$\sqrt{\frac{\rho_s}{\rho_f}} = \sqrt{\frac{Radius}{Thickness\ of\ the\ shell}} \qquad (23)$$

Rather than solving a 6th or higher order differential equations or using the software, which also takes significant computational time till date, a simple equation for a specific system is more convenient in structural engineering perspective. If the problems of fluid dynamics are not of primary interest which is the case for most of the structural engineering systems, the efficient way is to deal with these approximations of a given system if the error is not significant. This objective is served by the equation [23] for the cylinders filled with water. The equation [24] can be regarded as the apparent transition condition to differentiate strong and weak coupling of the cylinder structure with fluids. Hence this report will serve as a guide for the further parametric analysis on coupling in the similar systems, and to briefly understand the coupling factor and where and how to start the study. These formulations find applicability in designing the thin structures containing dense fluid (water) and evaluation of their modal characteristics, prediction of enclosure acoustics for fluid-filled structures.

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