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A STUDY ON A SUBCLASS OF INTUITIONISTIC PREOPEN SETS VIA INTUITIONISTIC GRILLS

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Abstract **Study of ideal and grill on a topological space is going on from1930 and 1947 respectively to till date. The aim of this paper is to introduce a class of intuitionistic sets in an intuitionistic topological space X, termed $\mathcal{I}_g$-open, which forms a subclass of the class of an intuitionistic pre open sets of X.**

Keywords: **Intuitionistic set, Intuitionistic grill, intuitionistic grill open .**

I. INTRODUCTION AND PRELIMINARIES

Intuitionistic sets and Intuitionistic points are introduced by D.Coker[4] in1996. This concept is originated from the study of Zadeh[6], who introduced Intuitionistic fuzzy sets in the year 1965.These sets are considered as the discrete form of Intuitionistic fuzzy sets and it is also one of several ways of introducing vagueness in mathematical objects. After Coker introduced Intuitionistic sets and topology based on these sets several papers were published in intuitionistic fuzzy topology. The idea of grill on a topological space was first introduced by Choquet [7]in 1947. Roy and Mukherjee[9], Noiri and Alomiri[10] have used grill on topological space as like ideals in topological space and have obtained many new topologies. It is observed from literature that the concept of grill is a powerful supporting tool, like nets and filters, in dealing with many topological concepts quite effectively. The idea of pre open sets was founded by Mashhour et al.,[11]. In this paper, we introduce and study a type of sets, defined in terms of intuitionistic grills. It is seen that such sets form asubclass of the class of all intuitionistic preopen sets.

Definition 1.1. [2] Let X be a nonempty fixed set. An intuitionistic set (IS for short) A is an object having the form A = < X, A^1, A^2> where A^1 and A^2 are subsets of X satisfying $A^1 \cap A^2 = \emptyset$. The set A^1 is called the set of members of A, while A^2 is called the set of non members of A. The collection of set of all intuitionistic subsets of the set X is denoted as IS P(X).

Definition 1.2. [2] Let X be a nonempty set. A = < X, A^1, A^2> and B= < X, B^1, B^2> be an intuitionistic sets on X and let $\{A_i: i \in J\}$ be an arbitrary family of intuitionistic sets in X, where A^i= < X, A_i^1, A_i^2>. Then

(a) $A \subseteq B$ if and only if $A^1 \subseteq B^1$and $B^2 \subseteq A^2$.
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
(c) $\cup A_i$= < X, $\cup A_i^1$, $\cap A_i^2$>.
(d) $\cap A_i$ = <X, $\cap A_i^1$, $\cup A_i^2$>.
(e)$\tilde{X}$ = < X, X, $\emptyset$>.
(f) $\tilde{\emptyset}$ = <X, $\emptyset$, X>.
(g) A^c= < X, A^2, A^1>.
(h) A – B = A ∩ B^c.

Definition 1.3. [4] Let X be a nonempty set and p ∈ X be a fixed element in X. Then $\tilde{p}$= <X, {p}, $\{p\}^c$> is called an intuitionistic point(IP for short)in X.

Definition 1.4. [2] Anintuitionistic topology (IT for short) on a nonemptyset X is a family τ of intuitionistic sets in X satisfying the following axioms:
(T_1) $\tilde{\emptyset}, \tilde{X} \in \tau$.
(T_2) $G_1 \cap G_2 \in \tau$ for any $G_1,\ G_2 \in \tau$.
(T_3) $\cup G_i \in \tau$ for any arbitrary family $\{G_i: i \in J\} \subseteq \tau$. In this case, the pair (X, τ) is called an intuitionistic topological space(ITS for short) and anyIS in $\tau$is known as an intuitionistic open set (IOS for short) in X.

Definition 1.5. [2] The complement A of an IOS A in an ITS (X, τ) is calledan intuitionistic closed set(ICS for short) in X.

Definition 1.6. [2] Let (X, τ) be an intuitionistic topological space andA = < X, A^1, A^2> be an IS in X. Then the interior and closure of A aredefined by
Iint(A) =∪{G : G is an IOS in X and G ⊆ A} and
ICl(A) = ∩{ K : K is ICS in X and A ⊆K}.

Definition 1.7. [8] A subcollection $\mathcal{I}_g$ (not containing the empty set) of ISP(X) is called an intuitionistic grill on X if $\mathcal{I}_g$satisfies the following conditions:

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(i) $A \in \mathcal{J}_g$ and $A \subseteq B \subseteq X$ implies $B \in \mathcal{J}_g$.

(ii) A, B $\subseteq$X and $A \cup B \in \mathcal{J}_g$ implies that either $A \in \mathcal{J}_g$ or $B \in \mathcal{J}_g$.

Example 1.8. [8] Let X = {a, b, c} and $\mathcal{J}_g$ = {< X, {a}, ∅>, < X, {c}, ∅>, < X, {a, b}, ∅>, < X, {a,c}, ∅>, < X, {b,c}, ∅>, $\tilde{X}$} be an intuitionistic grill on X.

Definition 1.9. [8] Let (X, τ) be an intuitionistic topological space and $\mathcal{J}_g$be an intuitionistic grill on X. We define a mapping Φ: ISP(X) → ISP(X),denoted by $\Phi_{\mathcal{J}_g}$(A, τ) for A ∈ ISP(X)) or $\Phi_{\mathcal{J}_g}$(A) called the operator associated with the intuitionistic grill $\mathcal{J}_g$ and the intuitionistic topology τ and it is defined by $\Phi_{\mathcal{J}_g}$(A) = {$\tilde{p}$ ∈X : A ∩ U ∈ $\mathcal{J}_g$for all U ∈ $\tau(\tilde{p})$}, where$\tau(\tilde{p})$ stands for the collection of all open neighbourhoods of $\tilde{p}$.

Result 1.10. [8] For a given intuitionistic grill $\mathcal{J}_g$on an intuitionistic topological space (X, τ), the map Φ : ISP(X) → ISP(X), $\psi_{\mathcal{J}_g}$(A) = A ∪ $\Phi_{\mathcal{J}_g}$(Ais a Kuratowski's closure operator giving rise to an intuitionistic topology $\tau_{\mathcal{J}_g}$(say)on X such that (a) $\tau \subseteq \tau_{\mathcal{J}_g}$ and (b) $\mathcal{B}(\mathcal{J}_g, \tau)$ = {V \ A: V ∈ τ and A ∉ $\mathcal{J}_g$} is an open base for$\tau_{\mathcal{J}_g}$.

Result 1.11. [8] Let $\mathcal{J}_g$ be an intuitionistic grill on an intuitionistic topologicalspace (X, τ). Then
(a) A ⊆B ⊆ X implies $\Phi_{\mathcal{J}_g}(A) \subseteq \Phi_{\mathcal{J}_g}(B)$.
(b) A ⊆ X and A ∉ $\mathcal{J}_g$ implies $\Phi_{\mathcal{J}_g}(A) = \tilde{\emptyset}$.
(c) $\Phi_{\mathcal{J}_g}(\Phi_{\mathcal{J}_g}(A)) \subseteq \Phi_{\mathcal{J}_g}(A) \subseteq$ ICl(A), for any A ⊆X.
(d) $\Phi_{\mathcal{J}_g}(A \cup B) = \Phi_{\mathcal{J}_g}(A) \cup \Phi_{\mathcal{J}_g}(B)$, for any A, B ⊆X.
(e) A ⊆ $\Phi_{\mathcal{J}_g}$(A) implies ICl(A) = $\tau_{\mathcal{J}_g}$-ICl(A) =ICl($\Phi_{\mathcal{J}_g}$(A)) = $\Phi_{\mathcal{J}_g}$(A).
(f) U ∈ $\tau$and $\tau \setminus \{\tilde{\emptyset}\} \subseteq \mathcal{J}_g$implies U ⊆ $\Phi_{\mathcal{J}_g}$(U).
(g) If U ∈ τ, then U ∩ $\Phi_{\mathcal{J}_g}$(A)= U ∩ $\Phi_{\mathcal{J}_g}$(U ∩ A), for any A ⊆ X.

Definition 1.12. [14] An intuitionistic subset A of an intuitionistic topologicalspace (X,τ) is called intuitionistic pre open(intuitionistic pre closed) if A ⊆IintIcl(A)(resp.IclIint(A) ⊆ A).
The intersection (union) of intuitionisticPre closed(resp. intuitionistic pre open) sets containing (resp. contained in) aset A is called the intuitionistic pre closure(resp. intuitionistic pre interior) ofA, to be denoted by Ipcl(A) (resp. Ipint(A)).

The set of all intuitionistic pre open sets in an intuitionistic topological space X is denoted by IPO(X).

Definition 1.13. [12] An intuitionistic subset A of an intuitionistic topologicalspace (X, τ) is called intuitionistic semi open if for some intuitionistic open setU, U ⊆ A⊆Icl(U).

The complement of an intuitionistic semi open set is called intuitionistic semi closed. The intersection of all intuitionistic semi closed sets containing A is called the intuitionistic semi closure of A and is denoted by Iscl(A).

Definition 1.14. [13] An intuitionistic subset A of an intuitionistic topologicalspace(X, τ) is called an intuitionistic semi-preopen if thereexists an intuitionistic pre open set U in X such that U ⊆ A ⊆ Icl(U).

The family of all intuitionistic semi preopen sets in (X, τ) is denoted by Ispo(X). For any A⊆ X, the union of all intuitionistic semi pre open sets contained in A is called an intuitionistic semi-preinterior of A and is denoted by Ispint(A).

Result 1.15. [15] An intuitionistic set A is intuitionistic semi closed if and only if A = Iscl(A).

Result 1.16. [13] For any intuitionistic set A of an intuitionistic topological space (X,τ), Iscl(A) = A ∪ IintIcl(A).

Definition 1.17. [14] An intuitionistic set A of an intuitionistic topologicalspace(X, τ) is called an intuitionistic regular open if A = Iint(Icl(A)).

II. INTUITIONISTIC GRILL OPEN SET

In this chapter we introduce $\mathcal{J}_g$-open sets, the definition being given interms of an intuitionistic grill on an intuitionistic topological space X and an operator $\Phi_{\mathcal{J}_g}$(A) introduced in [8].

Definition 2.1. Let $\mathcal{J}_g$ be an intuitionistic grill on an intuitionistic topologicalSpace (X, τ). An intuitionistic set A ⊆X is called $\mathcal{J}_g$-open if A ⊆Iint($\Phi_{\mathcal{J}_g}$(A)).

The complement of such an intuitionistic set is called $\mathcal{J}_g$-closed.

Example 2.2. Let X = {a, b, c} and
τ = { $\tilde{\emptyset}$, $\tilde{X}$, < X, {a}, {b,c}, < X, {b,c}, {a}>} be an intuitionistic topologyon X.
Consider an intuitionistic grill $\mathcal{J}_g$ ={< X, {a}, {b,c}>, < X, {c}, {a,b}>,< X, {a,b}, {c}>, < X, {a,c}, {b}>, < X, {a}, ∅>, < X, {a,b}, ∅>, < X, {a,c}, ∅>, < X, {a}, {b}>, < X, {a}, {c}>, < X, {b,c}, {a}>, < X, {c}, {a}>, < X, {c}, {b}>, < X, {c}, ∅>, < X, {b,c}, ∅>,$\tilde{X}$}
Let A =< X, {c}, {a,b}>.$\Phi_{\mathcal{J}_g}$(A) = {< X, {b}, {a,c}>, < X, {c}, {a,b}>} implies $\Phi_{\mathcal{J}_g}$(A) = < X, {b,c}, {a}>. Thus A is $\mathcal{J}_g$open. But not an intuitionistic open.

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Remark 2.3. Clearly, every $\mathcal{J}_g$ open set in any intuitionistic topological space (X, τ) is an intuitionistic preopen(by result 1.11(c)). The converse is not true.

Example 2.4. Let X = {a, b, c} and τ = { $\widetilde{\emptyset}$, $\tilde{X}$, < X, {a}, {b, c}>, < X, {b}, {a,c}>, < X, {a,b}, {c}>} be an intuitionistic topology on X. Consider an intuitionistic grill $\mathcal{J}_g$= { < X, {a}, {b, c}>, < X, {a,b}, {c}>,< X, {a,c}, {b}>, < X, {a}, {b}>, < X, {a}, {c}>, < X, {a}, ∅>, < X, {a,b}, ∅>, < X, {a,c}, ∅>, $\tilde{X}$}.
Let A = < X, {a,b}, {c}>. Then A is an intuitionistic open(hence an intuitionistic pre open)set in (X,τ). But $\Phi_{\mathcal{J}_g}$(A) = {< X, {a}, {b,c}>,< X, {c}, {a,b}>} implies $\Phi_{\mathcal{J}_g}$(A) = < X, {a,c}, {b}>, so that int($\Phi_{\mathcal{J}_g}$(A)) = < X, {a}, {b,c}>. Hence A is not $\mathcal{J}_g$ open.

Note 2.5. From previous two examples, intuitionistic open sets and $\mathcal{J}_g$-openSets are independent of each other.

Theorem 2.6. Let (X, τ) be an intuitionistic topological space and $\mathcal{J}_g$ -be anintuitionistic grillon X. Then arbitrary union of $\mathcal{J}_g$-open sets is $\mathcal{J}_g$-open.

Proof: Let {A_α : $\alpha \in$ J} be an intuitionistic family of $\mathcal{J}_g$-open subsets of(X, τ). Then $A_\alpha \subseteq$ Iint($\Phi_{\mathcal{J}_g}(A_\alpha)$) for each $\alpha \in$J. Now $\bigcup_{\alpha\in J} A_\alpha \subseteq$Iint($\Phi_{\mathcal{J}_g}(A_\alpha)$)⊆Iint($\Phi_{\mathcal{J}_g}(\bigcup_{\alpha\in J} A_\alpha)$)(by result 1.11(a)).

Remark 2.7. The intersection of two $\mathcal{J}_g$-open sets may not be $\mathcal{J}_g$-open set.

Example 2.8. Let X = {a, b, c} and τ = { $\widetilde{\emptyset}$, $\tilde{X}$, < X, {a,b}, {c}> }be an intuitionistic topology on X. Consider an intuitionistic grill $\mathcal{J}_g$ = { < X, {a}, {b,c}>, < X, {b}, {a,c}>, < X, {a,b}, {c}>,< X, {a,c}, {b}>, < X, {a}, {b}>, < X, {a}, {c}>, < X, {b}, {a}>, < X, {b}, {c}>, < X, {a, b}, ∅>,< X, {a,c}, ∅>, < X, {a}, ∅>, < X, {b}, ∅>,< X, {b,c}, ∅>,< X, {b,c}, {a}>, $\tilde{X}$}.
Let A = <X, {a,c}, {b}>implies$\Phi_{\mathcal{J}_g}$(A) = { < X, {a}, {b,c}>, < X, {b}, {a,c}>, < X, {c}, {a,b}>} implies $\Phi_{\mathcal{J}_g}$(A) = $\tilde{X}$, so that Iint ($\Phi_{\mathcal{J}_g}$(A)) = $\tilde{X}$ and let B= < X, {b,c}, {a}> implies$\Phi_{\mathcal{J}_g}$(B) = { < X, {a}, {b, c}>, < X, {b}, {a,c}>, < X, {c}, {a,b}>} implies $\Phi_{\mathcal{J}_g}$(B) = $\tilde{X}$. Iint($\Phi_{\mathcal{J}_g}$(B)) = $\tilde{X}$. Thus A and B are $\mathcal{J}_g$-open.Now A∩ B =< X, {c}, {a,b}>. But A ∩ B = < X, {c}, {a,b}>. But A ∩ B is not $\mathcal{J}_g$-open(as $\Phi_{\mathcal{J}_g}$(< X, {c}, {a,b}>) = $\widetilde{\emptyset}$).

Theorem 2.9. Let $\mathcal{J}_g$ be an intuitionistic grill on an intuitionistic topologicalspace (X, τ).

(a) If τ\ { $\widetilde{\emptyset}$} ⊆ $\mathcal{J}_g$, then every intuitionistic open set is $\mathcal{J}_g$-open set.

(b) If A ⊆X is $\mathcal{J}_g$-open and $\tau_{\mathcal{J}_g}$- intuitionistic closed, then A is an intuitionistic open.

Proof: (a) τ\ { $\widetilde{\emptyset}$} ⊆ $\mathcal{J}_g$ and U ∈ τ implies U⊆ $\Phi_{\mathcal{J}_g}$(U)(by result 1.11(f)). Now U = Iint U ⊆Iint(($\Phi_{\mathcal{J}_g}$(U)). Therefore U is $\mathcal{J}_g$-open.
(b) Since A is $\tau_{\mathcal{J}_g}$-intuitionistic closed, we have A =$\psi_{\mathcal{J}_g}$(A) = A ∪ $\Phi_{\mathcal{J}_g}$(A).
So that $\Phi_{\mathcal{J}_g}$(A) ⊆ A. Again, as A is $\mathcal{J}_g$-open, we have A ⊆Iint(($\Phi_{\mathcal{J}_g}$(A)) ⊆ Iint(A). Hence A is an intuitionistic open set.

Theorem 2.10. Let $\mathcal{J}_g$ be an intuitionistic grill on an intuitionistic topologicalspace (X, τ) .Then the following are equivalent.
(a) A is $\mathcal{J}_g$-open.
(b) A ⊆ $\Phi_{\mathcal{J}_g}$(A) and Iscl(A) = IintIcl(A).
(c) A ⊆ $\Phi_{\mathcal{J}_g}$(A) and A is an intuitionistic preopen.

Proof: (a) ⇒ (b) A is $\mathcal{J}_g$-open implies A ⊆ Iint(($\Phi_{\mathcal{J}_g}$(A)) implies A⊆ $\Phi_{\mathcal{J}_g}$(A). By result 1.11(c), A⊆ IintIcl(A). Then by using result 1.16, Iscl(A) = A ∪IintIcl(A) = IintIcl(A).

(b) ⇒(c) We have, A⊆ IsclA = IintIcl(A) implies A is an intuitionisticPre open.

(c) ⇒(a) Let A ⊆ $\Phi_{\mathcal{J}_g}$(A) and A is an intuitionistic pre open impliesA⊆ Iintcl(A). That impliesA ⊆ Iint(($\Phi_{\mathcal{J}_g}$(A))(since Icl(A) = $\Phi_{\mathcal{J}_g}$(A)). Therefore A is $\mathcal{J}_g$-open.

Corollary 2.11. Let $\mathcal{J}_g$ be an intuitionistic grill on an intuitionistic topological space (X, τ). If A⊆ X is intuitionistic semi closed and $\mathcal{J}_g$-open then A isan intuitionistic regular open.

Proof: Let A is $\mathcal{J}_g$-open. That implies Iscl(A) = IintIcl(A)(By Theorem2.10). Now A is an intuitionistic semi closed implies A = Iscl(A) = IintIcl(A).

Theorem 2.12. Let $\mathcal{J}_g$ be an intuitionistic grill on an intuitionistic topologicalspace (X, τ). If A ⊆X is $\mathcal{J}_g$ closed, then A ⊇IclIint(A) ⊇ $\Phi_{\mathcal{J}_g}$(IintA).

Proof: Let A is $\mathcal{I}_g$ closed implies $\tilde{X} \setminus$ A is $\mathcal{I}_g$-open implies $\tilde{X} \setminus$ A $\subseteq \Phi_{\mathcal{I}_g}(\tilde{X} \setminus$ A)(by Theorem 2.10). So by result 1.11(e), $\Phi_{\mathcal{I}_g}(\tilde{X} \setminus$ A) = Icl($\tilde{X} \setminus$ A)= $\tilde{X} \setminus$Iint(A). Thus $\tilde{X} \setminus$ A $\subseteq$ Iint ($\Phi_{\mathcal{I}_g}(\tilde{X} \setminus$ A)) implies$\tilde{X} \setminus$ A $\subseteq$ Iint($\tilde{X} \setminus$ Iint(A)) implies $\tilde{X} \setminus$ A = $\tilde{X} \setminus$IclIint(A) implies IclIint(A) $\subseteq$ A. ThusA$\supseteq$IclIint(A) $\supseteq \Phi_{\mathcal{I}_g}$(Iint(A))

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