A STUDY ON A SUBCLASS OF INTUITIONISTIC PREOPEN SETS VIA INTUITIONISTIC GRILLS

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Abstract Study of ideal and grill on a topological space is going on from 1930 and 1947 respectively to till date. The aim of this paper is to introduce a class of intuitionistic sets in an intuitionistic topological space X, termed $\mathcal{J}_g$-open, which forms a subclass of the class of an intuitionistic pre open sets of X.

Keywords: Intuitionistic set, Intuitionistic grill, intuitionistic grill open.

I. INTRODUCTION AND PRELIMINARIES

Intuitionistic sets and intuitionistic points are introduced by D.Coker[4] in 1996. This concept is originated from the study of Zadeh[6], who introduced intuitionistic fuzzy sets in the year 1965. These sets are considered as the discrete form of intuitionistic fuzzy sets and it is also one of several ways of introducing vagueness in mathematical objects. After Coker introduced Intuitionistic sets and topology based on these sets several papers were published in intuitionistic fuzzy topology. The idea of grill on a topological space was first introduced by Choquet [7] in 1947. Roy and Mukherjee[9], Noiri and Alomiri[10] have used grill on topological space as like ideals in topological space and have obtained many new topologies. It is observed from literature that the concept of grill is a powerful supporting tool, like nets and filters, in dealing with many topological concepts quite effectively. The idea of pre open sets was founded by Mashhour et al.[11]. In this paper, we introduce and study a type of sets, defined in terms of intuitionistic grills. It is seen that such sets form a subclass of the class of all intuitionistic preopen sets.

Definition 1.1. [2] Let X be a nonempty set. An intuitionistic set IS for short A is an object having the form A $\equiv$ $\langle X$, $A^1$, $A^2 \rangle$ where $A^1$ and $A^2$ are subsets of X satisfying $A^1 \cap A^2 = \emptyset$. The set $A^1$ is called the set of members of A, while $A^2$ is called the set of non members of A. The collection of set of all intuitionistic subsets of the set X is denoted as IS P(X).

Definition 1.2. [2] Let X be a nonempty set. A $\equiv$ $\langle X$, $A^1$, $A^2 \rangle$ and B $\equiv$ $\langle X$, $B^1$, $B^2 \rangle$ be an intuitionistic sets on X and let $\{A_i: i \in J \}$ be an arbitrary family of intuitionistic sets in X, where $A^i$ $\equiv$ $\langle X$, $A_i^1$, $A_i^2 \rangle$. Then

(a) $A \subseteq B$ if and only if $A^1 \subseteq B^1$ and $B^2 \subseteq A^2$.
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
(c) $\bigcup A_i$ $\equiv$ $\langle X$, $\bigcup A_i^1$, $\bigcup A_i^2 \rangle$.
(d) $\bigcap A_i$ $\equiv$ $\langle X$, $\bigcap A_i^1$, $\bigcap A_i^2 \rangle$.
(e) $\emptyset$ $\equiv$ $\langle X$, $\emptyset$, $X \rangle$.
(f) $\emptyset$ $\subseteq$ $\emptyset$.
(g) $A^c$ $\equiv$ $\langle X$, $A^2$, $A^1 \rangle$.
(h) $A - B$ $\equiv$ $A \cap B^c$.

Definition 1.3. [4] Let X be a nonempty set and $p \in X$ be a fixed element in X. Then $\bar{p}$ $\equiv$ $\langle X$, $\{p\}$, $\{p\}^c \rangle$ is called an intuitionistic point(IP for short) in X.

Definition 1.4. [2] An intuitionistic topology (IT for short) on a nonempty set X is a family $\tau$ of intuitionistic sets in X satisfying the following axioms:

(1) $\emptyset \in \tau$.
(2) $G_1 \cap G_2 \in \tau$ for any $G_1$, $G_2 \in \tau$.
(3) $\bigcup G_i \in \tau$ for any arbitrary family $\{G_i: i \in J \} \subseteq \tau$. This case, the pair $(X, \tau)$ is called an intuitionistic topological space(ITS for short) and any IS in X is known as an intuitionistic open set (IOS for short) in X.

Definition 1.5. [2] The complement $A$ of an IOS A in an ITS $(X, \tau)$ is called an intuitionistic closed set(ICS for short) in X.

Definition 1.6. [2] Let $(X, \tau)$ be an intuitionistic topological space and $A = \langle X$, $A^1$, $A^2 \rangle$ be an IS in X. Then the interior and closure of A are defined by

$\text{Int}(A) = \bigcup \{G: G$ is an IOS in X and $G \subseteq A\}$ and

$\text{Cl}(A) = \bigcap \{K: K$ is ICS in X and $A \subseteq K\}$.

Definition 1.7. [8] A subcollection $\mathcal{J}_g$ (not containing the empty set) of ISP(X) is called an intuitionistic grill on X if $\mathcal{J}_g$ satisfies the following conditions:
Definition 1.9. [8] Let \((X, \tau)\) be an intuitionistic topological space and \(I_{g}\) be an intuitionistic grill on \(X\). We define a mapping \(\Phi: ISP(X) \rightarrow ISP(X)\), denoted by \(\Phi_{g}(A, \tau)\) for \(A \in ISP(X)\) or \(\Phi_{g}(A)\) called the operator associated with the intuitionistic grill \(I_{g}\) and the intuitionistic topology \(\tau\) and it is defined by \(\Phi_{g}(A) = \{\varphi \in X: A \cap \varphi \in I_{g}\}\) for all \(A \in ISP(X)\), where \(\varphi\) stands for the collection of all open neighbourhoods of \(\varphi\).

Result 1.10. [8] For a given intuitionistic grill \(I_{g}\) on an intuitionistic topological space \((X, \tau)\), the map \(\Phi: ISP(X) \rightarrow ISP(X), \Psi_{g}(A) = A \cup \Phi_{g}(A)\) is a Kuratowski’s closure operator giving rise to an intuitionistic topology \(\tau_{g}\) (say) on \(X\) such that a) \(\tau \subseteq \tau_{g}\) and b) \(\tau_{g}(A, \tau) = \{V \in \tau: V \in \tau_{g}(A)\} \cup \tau_{g}(A, \tau)\) for all \(A \in ISP(X)\).

Definition 1.11. [12] An intuitionistic subset \(A\) of an intuitionistic topological space \((X, \tau)\) is called intuitionistic semi open if and only if \(A = I_{g}(A)\).

Definition 1.12. [14] An intuitionistic subset \(A\) of an intuitionistic topological space \((X, \tau)\) is called intuitionistic regular open if and only if \(A = I_{g}(A)\).

Definition 1.13. [12] An intuitionistic subset \(A\) of an intuitionistic topological space \((X, \tau)\) is called intuitionistic semi closed if and only if \(A = I_{g}(A)\).

Definition 1.14. [13] An intuitionistic subset \(A\) of an intuitionistic topological space \((X, \tau)\) is called an intuitionistic semi pre-open if there exists an intuitionistic semi pre-open set \(U\) in \(X\) such that \(U \subseteq A \subseteq I_{g}(U)\).

Definition 1.15. [15] An intuitionistic set \(A\) is intuitionistic semi closed if and only if \(A = Iscl(A)\).

Definition 1.16. [13] For any intuitionistic set \(A\) on an intuitionistic topological space \((X, \tau)\), \(Iscl(A) = A \cup I_{g}(A)\).

Definition 1.17. [14] An intuitionistic set \(A\) of an intuitionistic topological space \((X, \tau)\) is called an intuitionistic regular open if \(A = I_{g}(A)\).

II. INTUITIONISTIC GRILL OPEN SET

In this chapter we introduce \(I_{g}\)-open sets, the definition being given in terms of an intuitionistic grill on an intuitionistic topological space \(X\) and an operator \(\Phi_{g}(A)\) introduced in [8].

Definition 2.1. Let \(I_{g}\) be an intuitionistic grill on an intuitionistic topological space \((X, \tau)\). An intuitionistic set \(A \subseteq X\) is called \(I_{g}\)-open if \(A \subseteq I_{g}(A)\).

The complement of such an intuitionistic set is called \(I_{g}\)-closed.

Example 2.2. Let \(X = \{a, b, c\}\) and \(\tau = \{\emptyset, X, \{a, b, c\}, \{a, b\}, \{a\}, \{b\}, \{c\}, \emptyset\}\) be an intuitionistic topology on \(X\).

Consider an intuitionistic grill \(I_{g}\) on \(X\) such that \(I_{g}(a) = \{\emptyset, X, \{a, b, c\}, \{a, b\}, \{a\}, \{b\}, \{c\}, \emptyset\}\) and \(I_{g}(b) = \{\emptyset, X, \{a, b, c\}, \{a, b\}, \{a\}, \{b\}, \{c\}\}\).

Let \(A = X\) and \(B = \{a, b, c\}\)

Thus \(A\) is not \(I_{g}\)-open. But not an intuitionistic open.
Remark 2.3. Clearly, every $J_g$-open set in any intuitionistic topological space $(X, \tau)$ is an intuitionistic preopen(by result 1.11(c)). The converse is not true.

Example 2.4. Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ be an intuitionistic topology on $X$. Consider an intuitionistic grill $J_g= \{< X, \emptyset>, < X, a>, < X, b>, < X, \{a\}>, < X, \{b\}>, < X, \{a, b\}>, < X, \{a, c\}>, < X, \{b, c\}>, < X, \{a, b, c\}>\}$. Let $A = < X, \{a\}, \{b\}>$. Then $A$ is an intuitionistic open set.(hence an intuitionistic pre open set) set in $(X, \tau)$. But $\Phi_{J_g}(A) = < X, \emptyset>, < X, a>, < X, b>, < X, \{a\}>, < X, \{b\}>, < X, \{a, b\}>, < X, \{a, c\}, < X, \{b, c\}>, < X, \{a, b, c\}>, < X, \emptyset>, < X, \{a\}, \{b\}, < X, \{a, b\}>$, so that $\text{int}(\Phi_{J_g}(A)) = < X, \{a\}, \{b\}>$. Hence $A$ is not $J_g$-open.

Note 2.5. From previous two examples, intuitionistic open sets and $J_g$-open sets are independent of each other.

Theorem 2.6. Let $(X, \tau)$ be an intuitionistic topological space and $J_g$ be an intuitionistic grill on $X$. Then $X$ is an intuitionistic preopen if and only if $\phi(X) = \{< X, \emptyset>, < X, a>, < X, b>, < X, \{a\}>, < X, \{b\}>, < X, \{a, b\}>\}$.

Proof: Let $\{A_\alpha : \alpha \in J\}$ be an intuitionistic family of $J_g$-open subsets of $(X, \tau)$. Then $A = \bigcup_{\alpha \in J} \text{int}(\Phi_{J_g}(A_\alpha))$ for each $\alpha \in J$. Now $\bigcup_{\alpha \in J} A_\alpha \subseteq \text{int}(\Phi_{J_g}(A)) \subseteq \text{int}(\Phi_{J_g}(\bigcup_{\alpha \in J} A_\alpha))$ (by result 1.11(a)).

Remark 2.7. The intersection of two $J_g$-open sets may not be $J_g$-open.

Example 2.8. Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ be an intuitionistic topology on $X$. Consider an intuitionistic grill $J_g = \{< X, \emptyset>, < X, a>, < X, b>, < X, \{a\}>, < X, \{b\}>, < X, \{a, b\}>\}$. Let $A = < X, \{a\}, \{b\}>$. Then $A$ is an intuitionistic open set.(hence an intuitionistic pre open set) set in $(X, \tau)$. But $\Phi_{J_g}(A) = < X, \emptyset>, < X, a>, < X, b>, < X, \{a\}>, < X, \{b\}, < X, \{a, b\}>$, so that $\text{int}(\Phi_{J_g}(A)) = < X, \{a\}, \{b\}>$. Hence $A$ is not $J_g$-open.

Theorem 2.9. Let $J_g$ be an intuitionistic grill on an intuitionistic topological space $(X, \tau)$.

(a) If $\forall \alpha \in J$ then every intuitionistic open set is $J_g$-open.

(b) If $A \subseteq X$ is $J_g$-open and $\tau_{J_g}$-intuitionistic closed, then $A$ is an intuitionistic open.

Proof: (a) $\forall \alpha \in J$ then $U \subseteq \text{int}(\Phi_{J_g}(U))$ (by result 1.11(f)). Now $U = \bigcup_{\alpha \in J} \text{int}(\Phi_{J_g}(U))$. Therefore $U$ is $J_g$-open.

(b) Since $A$ is $\tau_{J_g}$-intuitionistic closed, we have $A = \Phi_{J_g}(A) = A \cup \Phi_{J_g}(A)$. So that $\Phi_{J_g}(A) \subseteq A$. Again, as $A$ is $J_g$-open, we have $A \subseteq \text{int}(\Phi_{J_g}(A)) \subseteq \text{int}(A)$. Hence $A$ is an intuitionistic open set.

Theorem 2.10. Let $J_g$ be an intuitionistic grill on an intuitionistic topological space $(X, \tau)$. Then the following are equivalent.

(a) $A$ is $J_g$-open.

(b) $A \subseteq \Phi_{J_g}(A)$ and $\text{Iscl}(A) = \text{Icl}(A)$.

(c) $A \subseteq \Phi_{J_g}(A)$ and $A$ is an intuitionistic preopen.

Proof: (a) $\Rightarrow$ (b) $A$ is $J_g$-open implies $A \subseteq \text{Icl}(\Phi_{J_g}(A))$ implies $A \subseteq \Phi_{J_g}(A)$. By result 1.11(c). $\text{Icl}(A) = \text{Icl}(A)$ by using result 1.16, $\text{Icl}(A) = \text{Icl}(A) \cup \text{Icl}(A)$. Therefore $A$ is $J_g$-open.

(b) $\Rightarrow$ (c) We have, $A \subseteq \text{Icl}(A)$ implies $A$ is an intuitionistic preopen.

(c) $\Rightarrow$ (a) Let $A \subseteq \Phi_{J_g}(A)$ and $A$ is an intuitionistic pre open implies $A \subseteq \text{Icl}(A)$. That implies $A \subseteq \text{Icl}(\Phi_{J_g}(A))$ since $\text{Icl}(A) = \Phi_{J_g}(A)$. Therefore $A$ is $J_g$-open.

Corollary 2.11. Let $J_g$ be an intuitionistic grill on an intuitionistic topological space $(X, \tau)$. If $A \subseteq X$ is intuitionistic semi closed and $J_g$-open then $A$ is intuitionistic regular open.

Proof: Let $A$ is $J_g$-open. That implies $\text{Icl}(A) = \text{Icl}(A)$ (by Theorem 2.10). Now $A$ is an intuitionistic semi closed implies $A = \text{Iscl}(A) = \text{Icl}(A)$.

Theorem 2.12. Let $J_g$ be an intuitionistic grill on an intuitionistic topological space $(X, \tau)$. If $A \subseteq X$ is $J_g$ closed, then $A \supseteq \text{Icl}(A) \supseteq \Phi_{J_g}(\text{Icl}(A))$. 

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**Proof:** Let $A$ is $J_g$ closed implies $X\setminus A$ is $J_g$-open implies $X\setminus A \subseteq \Phi_g(X\setminus A)$ (by Theorem 2.10). So by result 1.11(e),

\[ \Phi_g(X \setminus A) = \text{Icl}(X \setminus A) = \text{Int}(\text{Icl}(X \setminus A)). \]

Thus $X \setminus A \subseteq \text{Int} \left( \Phi_g(X \setminus A) \right)$ implies $X \setminus A \subseteq \text{Int}(X \setminus \text{Int}(A))$ implies $X \setminus A = X \setminus \text{Int}(A)$ implies $\text{Icl}(X \setminus A) \subseteq A$. Thus $A \subseteq \text{Icl}(X \setminus \text{Int}(A)) \subseteq \Phi_g(\text{Int}(A)).$

### III. REFERENCES


