ON THE ANALYSIS OF NAVIER SLIP AND NEWTONIAN HEATING ON STEADY MAGNETOHYDRODYNAMIC REACTING CONVECTIVE FLUID FLOW OVER A VERTICAL PLATE

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Abstract—Effects of Navier Slip and Newtonian Heating on steady magnetohydrodynamic reacting convective flow over a vertical surface are considered in this paper. Similarity transformation technique is applied to transform the partial differential equations to coupled non-linear ordinary differential equations corresponding to momentum and energy equations. The corresponding coupled non-linear ordinary differential equations are solved numerically using shooting method along with fourth-order Runge-Kutta scheme. The results are presented in tables and graphs. The velocity and temperature of the flow increase with an increase in both local slip parameter and local Biot number.

Keywords—Navier slip, Newtonian heating, Steady, Magnetohydrodynamic, Convective flow, Biot number.

I. INTRODUCTION

The field of magnetohydrodynamics has caught the attention and interest of many scholars and researchers in Science, Engineering and Medicine ever since its discovery in 1918. Its practical applications are found in the design of fusion reactors, dispersion of metals, metallurgy, MHD pumps, MHD generator, MHD flow meter etc. So many reviews on topics like dispersion of metals, metallurgy, MHD engineering and medicine have been presented in tables and graphs. The velocity and temperature of the flow increase with an increase in both local slip parameter and local Biot number.

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increases with an increase in the Eckert number \((E_c)\). Winifred and Makinde (2013) studied combined effects of Buoyancy force and Navier slip on MHD flow of a nanofluid over a convectively heated vertical porous plate. They considered steady laminar incompressible two-dimensional MHD boundary layer flow of an electrically conducting water-based nanofluid past a convectively heated porous vertical semi-infinite flat plate under combined effects of buoyancy forces and Navier slip. It was found that (i) the fluid velocity increases with increase in the slip parameter, (ii) Magnetic parameter causes a decrease in the fluid velocity, (iii) Grashof and Eckert numbers cause a decrease in the velocity profile, and (iv) increase in the intensity of Newtonian heating and Eckert number causes an increase in the temperature of the fluid. In a recent research carried out by Fagbade Falodun, Omowaye (2018) on MHD natural convection flow of viscoelastic fluid over an accelerating permeable surface with thermal radiation and heat source or sink, it was revealed that an increase in the parameter of the flow produces significant increase in the fluid temperature.

Yahaya (2016) studied steady MHD boundary-layer slip flow and heat transfer of nanofluid over a convectively heated non-linear permeable sheet. A two-dimensional steady boundary layer flow of nanofluid over a non-linear permeable stretching sheet with partial slip was considered. With some necessary assumptions, the coupled non-linear higher order ordinary differential equations obtained were solved, the results of which revealed that increase in Biot number increases with an increase in the parameter of the flow produces significant increase in the intensity of Newtonian heating and Eckert number causes an increase in the temperature of the fluid. In a recent research carried out by Fagbade Falodun, Omowaye (2018) on MHD natural convection flow of viscoelastic fluid over an accelerating permeable surface with thermal radiation and heat source or sink, it was revealed that an increase in the parameter of the flow produces significant increase in the fluid temperature.

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Anuradha and Priyadharshini (2014) studied unsteady free convective MHD flow of a viscous fluid past a semi-infinite vertically moving porous plate embedded in a porous medium. Makinde and Ibrahim (2013) investigated the MHD chemically reacting viscous fluid flow towards a vertical surface with slip and convective boundary condition. It was established that (i) the magnetic field strength increases the flow of fluid towards the surface, (ii) the Grashof number has positive influence on the flow field, and (iii) the rate of heat transfer diminishes with increasing Grashof, magnetic and Prandtl numbers. Chand and Jat (2014) studied the effects of viscous dissipation and radiation on steady flow of electrically conducting fluid through a porous stretching surface. The expressions of the velocity and temperature were found with classical Runge-kutta fourth-order scheme. Mathews and Hills (2008) investigated numerically the effect of slip on momentum boundary layer thickness on a flat plate. The combined effects of Hall current and wall slip on steady MHD flow of a visco-elastic fluid past an infinite vertical porous plate through a porous medium was investigated by Kumar and Chand (2011).

The main goal of this work is to determine the combined effects of Navier slip and Newtonian heating on steady magnetohydrodynamic reacting convective flow over a vertical surface. Subsequently, the governing partial differential equations of the flow are transformed into ordinary differential equations using appropriate similarity variables which are later solved numerically using the shooting method along with fourth order Runge-Kutta scheme. Conclusively, the results are presented in tables and graphs. The discussion of which is accompanied with some concluding remarks.

II. MATHEMATICAL MODEL

We consider a steady two-dimensional flow of a viscous incompressible flowing along a vertical plate at constant temperature under Arrhenius kinetics. As indicated in figure 1 below. We assumed that: (i) the flow is in the x direction along the plate to which the y axis is perpendicular, (ii) a uniform strong magnetic field \(β_o\) is applied normal to the flow, (iii) the flow is not influenced by any external field due to polarization of charges and Hall effect; hence, the electrical field is considered to be negligible although there is heat generation due to viscous dissipation. Also, the assumption of no slip condition on the flow is not valid rather the flow is under the influence of Navier slip and Newtonian heating.

![Figure 1. The schematic diagram of the flow](image)

Taking into consideration the Boussinesq’s approximation, the equations governing the flow corresponding to continuity, momentum and energy respectively are written as follows:
Frank Kametniskii parameter: \( \delta = \frac{AQe}{U_c \rho} \).

Activation energy parameter: \( e = \frac{RT_{\infty}}{\varepsilon} \).

Local Biot number: \( Bi_i = \frac{h}{k \sqrt{U_c}} \).

Local slip parameter: \( D = \beta \frac{U_c}{\varepsilon} \).

\( (\text{in this work, } \alpha = 1) \)

### III. NUMERICAL COMPUTATIONS

The set of nonlinear coupled ordinary differential equations (9) and (10) governing the flow of the fluid under the boundary conditions (11) and (12) are numerically solved using Runge-Kutta fourth order scheme alongside with shooting method. The step size of 0.01 is used for the numerical computation with 10-place decimal accuracy as the criterion of convergence. The results are presented in tables and figures. Verifying the accuracy of this present work, a direct comparison of the result obtained is done with the result obtained by Makinde and Olanrewaju [10] and this is shown in table 1.

### IV. RESULTS AND DISCUSSIONS

The problem of Navier slip and Newtonian heating on steady magnetohydrodynamic reacting convective flow over a vertical plate has been carried out in this work. Numerical computations for the nonlinear coupled ordinary differential equations governing the flow for different values of the parameters employing the numerical method were discussed in section III. In this section, discussions on the effects of the controlling parameters on the flow are presented. Meanwhile, all through the computations, the various parameters controlling the flow such as Eckert number (Ec), local Eckert number (Ec_i), Magnetic parameter (M), local Biot number (Bi), Slip parameter (D), among others, were varied at intervals for both velocity and temperature profiles. Figure 2 shows the velocity profiles for various values of local Grashof number \( (Gr) \). An increase in \((Gr)\) causes the velocity to increase which shows that buoyancy force assists the flow and...
this is in agreement with what Omowoye and Koriko (2014) obtained. Figure 3 shows the influence of magnetic parameter on the velocity distribution. The application of magnetic field to an electrically conducting fluid gives rise to a force called Lorentz force (resistance force) that causes retardation in the motion of the fluid hence reducing the velocity. The retarding force (Lorentz force) can be used to control the velocity of the flow. The influence of dissipation parameter, Ec, on the velocity profiles is presented in figure 4. It is observed that increase in Eckert number (Ec) encourages heat storage which in turn increases the kinetic energy of the fluid molecules; the effect of which causes an increase in the velocity of the fluid. Figure 5 illustrates the effect of Prandtl number (Pr) on the temperature profiles. It is observed that increase in Pr causes a decrease in temperature of the fluid. This is so because small value of Prandtl number (Pr), the lesser the thermal conductivity of the fluid becomes. The effect of Eckert number (Ec) on the temperature profile is shown in figure 6. It is noticed that as Eckert number (Ec) increases, the temperature increases. With the increase in Eckert number, heat energy is stored in the fluid due to the frictional drag. This is also in agreement with what Ramzan et al. [7] obtained. The influence of Magnetic field parameter (M) on temperature is shown in figure 7. It is observed that as the magnetic parameter increases so is the temperature of the fluid on the increase. Increase in M causes the temperature of the fluid to increase because the introduction of magnetic field to an electrically conducting fluid will give rise to Lorentz force which will oppose the motion of the fluid. Hence, the kinetic energy of the molecules of the fluid will be converted to heat energy in an irreversible process of viscous dissipation and this will give rise to temperature increase. Figure 8 shows that increase in Prandtl number (Pr) causes a decrease in the velocity of the flow. The increase in the Prandtl number is responsible for increase in the viscosity of the fluid which in turn opposes the motion of the fluid. Figure 9 shows that as the slip parameter (D) increases, so does the fluid velocity. This is because due to the slip condition at the plate, the velocity of the fluid adjacent to the plate permits more fluid to slip past the surface; hence, the flow accelerates for distances close to the surface. This is consistent with the fact that higher D means an increase in lubrication and slipperiness of the surface. In figure 10, the effect of slip parameter is illustrated on temperature. It is observed that an increase in the slip parameter causes a decrease in the temperature profiles of the flow. The presence of slip condition on the surface enables more fluid to flow past it. Hence, fluid molecules move more freely and there will be less viscous dissipation which can give rise to temperature rise. In figure 11, it is observed that an increase in the Newtonian heating parameter (Bi,) increases the fluid velocity. This is because the local Biot number (Bi,) enhances heat conduction resistance in the solid body. Hence, heat convection takes place between the body and the fluid. The transferred heat weakens the cohesive forces among the fluid’s molecules and this allows them to move faster. Figure 12 shows that an increase in the Newtonian heating causes a temperature rise in the fluid. Biot number increases thermal conductivity of the fluid which will enable the fluid to absorb more heat from the plate and this in turn will increase the temperature of the fluid.

V. GRAPHS

![Figure 2: Velocity profiles at varied Grs when Pr = 0.72, M = Ec = Ec, = 0.01, ε = 0.20, δ = 0.03, D = 0.10, Bi, = 0.10](image2)

![Figure 3: Velocity profiles at varied M when Pr = 0.72, Ec = Ec, = 0.01, ε = 0.20, δ = 0.03, D = 0.10, Bi, = 0.10, Gr, = 0.30](image3)

![Figure 4: Velocity profiles at varied Ec when Pr = 0.72, M = 0.01, ε = 0.20, δ = 0.03, D = 0.10, Bi, = 0.10, Gr, = 0.30](image4)
Figure 5: Temperature profiles at varied $Pr$ when 
$M = Ec = Ec_c = 0.01, e = 0.20, \delta = 0.03, D = 0.10, Bi_i = 0.10, Gr_r = 0.30$

Figure 8: Velocity profiles at varied $Pr$ when 
$Gr_r = 0.30, M = Ec = Ec_c = 0.01, e = 0.20, \delta = 0.03, D = 0.10, Bi_i = 0.10$

Figure 6: Temperature profiles at varied $Ec$ when 
$Pr = 0.72, M = 0.01, e = 0.20, \delta = 0.03, D = 0.10, Bi_i = 0.10, Gr_r = 0.30$

Figure 9: Velocity profiles at varied $D$ when 
$Gr_r = 0.30, M = Ec = Ec_c = 0.01, e = 0.20, \delta = 0.03, Pr = 0.72, Bi_i = 0.10$

Figure 7: Temperature profiles at varied $M$ when 
$Pr = 0.72, Ec = Ec_c = 0.01, e = 0.20, \delta = 0.03, D = 0.10, Bi_i = 0.10, Gr_r = 0.30$

Figure 10: Temperature profiles at varied $D$ when 
$Gr_r = 0.30, M = Ec = Ec_c = 0.01, e = 0.20, \delta = 0.03, Pr = 0.72, Bi_i = 0.10$
The effects of Navier slip and Newtonian heating on steady magnetohydrodynamic reacting convective flow over a vertical plate have been investigated. The concepts of Navier slip and Newtonian heating are examined and analyzed. The solutions of the coupled non-linear ordinary differential equations are computed using the fourth order Runge-Kutta scheme together with shooting method. The increasing effects of both the slip parameter and Biot number are felt as they both have increasing effects on both velocity and temperature of the fluid.

VI. CONCLUSION

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VIII. REFERENCE


Figure 11: Velocity profiles at varied $Bi$, when $Gr = 0.30, M = Ec = Ec, = 0.01, \epsilon = 0.20, \delta = 0.03, D = 0.10, Pr = 0.72$

Figure 12: Temperature profiles at varied $Bi$, when $Gr = 0.30, M = Ec = Ec, = 0.01, \epsilon = 0.20, \delta = 0.03, D = 0.10, Pr = 0.72$


